

# **M E T A P H Y S I C A**

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## **ARTICLES**

- CHHANDA CHAKRABORTI  
Mental Properties and Levels of Properties 7
- AMIHUD GILEAD  
A Possibilist Metaphysical Reconsideration  
of the Identity of Indiscernibles and Free Will 25
- SORIN BANGU  
Later Wittgenstein on Essentialism,  
Family Resemblance and Philosophical Method 53
- CARLOS DUFOUR  
Identity and Predication  
Observations on P. Monaghan's Thesis 75
- URIAH KRIEGEL  
Tropes and Facts 83
- MARTIN COOKE  
To Continue with Continuity 91

## **REVIEWS**

- Stephen Neale: *Facing Facts*  
Oxford: Clarendon Press 2001  
*by Herbert Hochberg* 111

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# ARTICLES

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CHHANDA CHAKRABORTI

## **Mental Properties and Levels of Properties**\*

### ABSTRACT

John Heil, independently and with David Robb, has recently proposed a non-traditional conception of properties. This ontology of properties does not allow any higher or lower level or order of being among the properties. Heil and Robb have claimed that their ontology of properties can solve most of the problems in philosophy of mind, because most of these problems are based on a faulty conception of the mental property. They also claim that from their ontology as a consequence it follows that the mental properties are physical properties and we need not introduce the mental property as a distinct property.

This paper argues that their arguments and ontological precepts may show that it is possible to do without a view of mental properties as a higher level property, but thereby they do not also show that it is possible to do without the mental property as a distinct property. It also argues that introduction of distinct property layers need not be the only option available for an anti-reductionist interested in doing metaphysics of mental properties. An anti-reductionist may defend the irreducibility claim of the mental as a distinct property without endorsing the ontology of properties that Heil and Robb find so objectionable. So, the rejection of a layered conception of properties in general need not imply rejection of the claim of the mental as a distinct property.

John Heil, individually<sup>1</sup> and also with David Robb<sup>2</sup>, has recently proposed a somewhat non-traditional ontology of properties which allows for no lev-

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els, higher or lower, of being in reality. Through a critique of what they claim is a more commonly held theory of properties, Heil and Robb have tried to raise questions about the tenability of the metaphysical presuppositions underlying the notion of the mental properties in current theories in philosophy of mind. Their criticisms, if valid, imply that many controversies in philosophy of mind are founded on a misconception about the mental properties and prevalent theories about the mental properties, such as property dualism, are not ontologically correct.

This article is an attempt of an assessment of the claims of this ontology vis-à-vis the mental properties. I argue that their arguments for ontological eradication of the mental property as a higher level property does not entail the ontological abolition of the mental as a distinct property. To think that it does is to conflate between what forms the core in the conceptualization of the mental property in the anti-reductionist theories such as property dualism and what could be deployed by some defenders as an explanatory framework around that core. I contend that the arguments of Heil and Robb are directed towards the latter, and do not touch the former. I end the article with some suggestions about how a theory of the mental can sustain its anti-reductionist character without subscribing to the ontology that Heil and Robb have found objectionable.

### *Section 1. Property Dualism as an example of anti-reductionism*

It is true that contemporary philosophy of mind is replete with talks about the mental properties. For example, for property dualism<sup>3</sup>, which has come to be accepted as a major choice as an anti-reductionist metaphysical alternatives, this notion is pivotal. Property dualism, as a position, claims that

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<sup>1</sup> Heil, John. *From an ontological point of view*. (Oxford: Clarendon Press, 2003). Henceforth in this article referred to as Heil 2003.

<sup>2</sup> Heil, John, and David Robb. "Mental Properties". *American Philosophical Quarterly* 40 No.3 (2003): 175-196. Henceforth in this article referred to as Heil and Robb 2003.

<sup>3</sup> Sometimes a cluster of theories are considered under the heading 'Property Dualism'. This is how Paul M Churchland, for instance, approaches the topic of property dualism (see Churchland, 1993, p.10). On the assumption that the diversity in this cluster comes from further additions of details resulting in different versions within the position, in this article I have taken a singular approach. I have referred to property dualism as a certain kind of metaphysical position which allows differences within the position.

though there is no separate substance as the mental substance, there are two basic kinds of properties in the world, the physical (e.g. *having a mass*) and the mental (e.g. *being a belief*, or *being a desire*). In other words, it is held that the objects in the world are fundamentally physical by nature, but under suitable conditions they can have (at most) two different kinds of properties, the physical and the mental. Both kinds of properties are considered to be real and are held as *being not reducible* to each other in the sense of being *different* from each other in some putative sense.

This class distinction between two kinds of properties, which is often labeled as type-dualism in recent literature, is also present in Cartesian substance dualism. In that scheme, however, the type dualism carves reality up into two neat halves. Two entirely different sets or *kinds* of properties or features are supposed to characterize the two different substances, affirming and explaining the essential difference that is supposed to exist between the two kinds of substance. Each exclusive set of properties requires a completely different kind of substance for instantiation.

The type-dualism supported by property dualism is definitely different from this. The type distinction between its two kinds of properties is not a consequence of a corresponding difference at the substance level. Moreover, property dualism allows that two different kinds of properties can be instantiated or co-instantiated in the same physical entity. As for example, a human being can have the physical property of *is 55 Kg (in weight)*, and the mental property of *is a belief that Santa Claus is real*. In fact, the challenge for property dualism is to show how well its ontology can accommodate unexceptionally physical objects with a dualistic division among the properties which characterize these objects. Its critics believe that this uncomfortable metaphysical situation either makes the mental causally impotent towards behavior and leads to epiphenomenalism, or results in causal overdetermination and go against the principle of metaphysical economy. Others<sup>4</sup> do not think so.

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<sup>4</sup> See for example Jackson, Frank. "Epiphenomenal qualia". *Philosophical Quarterly* 32, No.127 (1982): 127-136; Mills, Eugene. "Interaction and overdetermination", *American Philosophical Quarterly*, 33 (1996): 105-15.

Though it is easy to confuse it with predicate dualism, property dualism is not just another name for predicate dualism. While predicate dualism remains satisfied with the claim that the physical and the mental are merely *two different ways of characterizing* the essentially same physical thing, property dualism goes one step further to claim that the two *different types of properties* are the two types of characteristics that the objects really have. Its claims uses the following metaphysical assumption as the backdrop:

*1. Properties, as characteristics of objects, exist*

Different versions of property dualism<sup>5</sup> have emerged which employ different kinds of arguments in support of their thesis. But on the whole, a property dualist seems to favor a certain degree of realism, as is compatible with the different accounts of properties that envisage them as ‘something that is really out there’ and not merely existing as predicates.

**Irreducibility claim:** However, the most distinctively different claim of property dualism as an anti-reductionist theory is that *mental properties exist*. If in the context of dualistic division, the property of *being non-physical* may be taken as coextensive of the property of *being mental*, then we can formulate this important claim of a property dualist as follows:

2. There exists at least one property  $x$  such that  $x$  is not a physical property.  $[(\exists x) (\sim Px)]$ , where the universe of discourse is of properties, and  $Px$  stands for *x is physical*]

Alternatively, if it is not acceptable to take the property of *being mental* as coextensive to the property of *being non-physical*, the claim may be stated as:

2'. There exists at least one property  $x$  such that  $x$  is a mental property.  $[(\exists x) (Mx)]$ , where the universe of discourse is of properties, and  $Mx$  stands for *x is mental* ]

I shall refer to this claim as the *irreducibility claim*. 2 or 2' is an unequivocal assertion of the existence of the mental as a property distinct from the

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<sup>5</sup> See for example Churchland, Paul, M. *Matter and Consciousnesses: A Contemporary Introduction to the Philosophy of Mind*. (Cambridge, Massachusetts: The MIT Press, 1993 5<sup>th</sup> Printing), 10-13.



physical property. Unless this claim is held, as I see it, property dualism cannot be distinguished very well from its physicalistic or reductionist counterparts in philosophy of mind. For, as mentioned earlier, property dualism accepts that at the substance level everything is fundamentally physical. Churchland<sup>6</sup> asserts that this important claim identifies the position as dualist. I take the *irreducibility claim* as a core commitment to anti-physicalism or anti-reductionism. It, for example, will form the core of a bare minimum version of property dualism. Heil and Robb suggest that their ontology makes this irreducibility claim entirely redundant. I disagree.

**Higher and lower levels of properties claim:** Discussions in contemporary philosophy of mind often contain a reference to *levels* or *layers of properties*. This does not mean merely that the level of properties is different from the level of the things which they characterize. Different levels are said to exist among the properties. Microphysical properties, neurobiological events and properties in the brain etc. are often supposed to be *lower level* properties. The mental properties and complex physical properties, on the other hand, are unexceptionally said to be *higher level*<sup>7</sup> properties.

Schaffer<sup>8</sup>, for instance, cites a “standard” view of properties which he attributes to Newton to start with and also to contemporary philosophers such as Putnam, Kim, and Fodor<sup>9</sup>. On this view, the properties and the associated sciences are seen as arranged in layers and each higher layer is supposed to supervene on the lower layer. Schaffer describes it as follows:

It is now standard to think of nature as *layered* on which the natural properties are ordered into supervenience families: mental properties, which then

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<sup>6</sup> Ibid., 12.

<sup>7</sup> See for example Chalmers, David. *The Conscious Mind: In Search of a Fundamental Theory*. (Oxford: Oxford University Press 1996.); Kim, J. *Supervenience and the Mind: Selected Philosophical essays*. (Cambridge: Cambridge University Press, 1993).

<sup>8</sup> Schaffer, Jonathan. “Two conceptions of sparse properties”, *Pacific Philosophical Quarterly* 85 (2004): 92-102.

<sup>9</sup> Fodor, J. “Special sciences and the disunity of science as a working hypothesis”, *Synthese*, 28 ((1974):, 77-115.

supervene upon chemical properties, atomic properties, particle properties, quark properties, and perhaps more below. The levels of nature are reflected in the hierarchy of science: psychology, which is above biology, which is then above chemistry, atomic physics, particle physics, quark physics, and perhaps more below<sup>10</sup>.

Heil and Robb interpret those, who place the considerations about the mental property within a theory of higher and lower levels, to assume the following:

3. Higher level properties exist and the mental property is one of them.

They cite<sup>11</sup> Putnam and Fodor are to subscribe the view that the same creature can have both the *higher* level property pain and some lower level physical property as the realizer.

In their ontology, Heil and Robb are particularly critical of this layered view of properties, which they claim assume levels of reality. They maintain that it is a fiction created out of false metaphysical expectations.

How does their criticism pertain to the discussion of the mental property in anti-reductionist theories? Heil and Robb appear to think that their ontological criticisms affect it negatively. They suggest that their arguments against the layered view of properties and in favor of a no-layer ontology also show that there is no need to accept the mental as a distinct property. I disagree. In Section 3 of this paper, I argue that their ontological precepts may show that it is possible to do without a view of mental properties as a higher level property, but thereby they do not also show that it is possible to without the mental property as a distinct property. Moreover, in Section 4, I try to show that 3 need not be the only option available for an anti-reductionist interested in the metaphysics of mental properties. So, rejection of 3 in general need not imply rejection of 2 or 2'. But first, in Section 2, I present a brief summary of the ontology proposed by Heil (2003) and Heil and Robb (2003).

## 2. *An Alternative Conception of Property*

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<sup>10</sup>Schaffer, Jonathan, 2004, 92.

<sup>11</sup> Heil and Robb (2003), 179.

Heil and Robb (2003) maintain that an ontology of a hierarchically arranged levels of being among the properties owes its existence to some profound misconceptions about the nature of a property. Heil (2003) considers it also at the root of many contemporary philosophical conundrums. As he puts it,

In leaving behind levels, we leave myriad philosophical puzzles. These, if I am right, are puzzles of our own making<sup>12</sup>.

In particular, they claim that abandonment of the notion of hierarchical layers of properties will resolve some of the most vexing controversies in philosophy of mind. According to them, while espousing doctrines about the mental property, recent philosophers of mind should have settled, in particular, their ontology of properties first<sup>13</sup> and then they would have avoided many of the difficulties.

There are well-known “difficult disputes”<sup>14</sup> in metaphysics about how properties are to be conceived. In each of these disputes, Heil and Robb take what they call a non-traditional position. They reject three following widely held doctrines about properties:

- A. *Predicates are related to properties by correspondence*
- B. *Properties are universals*
- C. *Properties are either categorical or dispositional but not both*

In their ontology, properties and predicates are different. Properties are viewed as *the ways a particular object is*. Predicates help to express properties, however, in this ontology in order to be meaningful, every distinct predicate does not have to have a corresponding property that it uniquely designates or names. A predicate may apply to an object, not by virtue of the unique property that it names, but by virtue of some property. It may apply by virtue of salient similarities or resemblances, exact or ranging between *more to less*<sup>15</sup> among certain objects. They say that they also pre-

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<sup>12</sup> Heil (2003), 8.

<sup>13</sup> Heil and Robb (2003), 190.

<sup>14</sup> Armstrong, D.M. *Truth and Truthmakers*. (Cambridge, UK: Cambridge University Press, 2004), 43.

<sup>15</sup> Heil and Robb (2003), 183.

fer a “sparse”<sup>16</sup> and *in re* notion of properties, favoring only those properties which are determined to exist “by our best scientific effort”<sup>17</sup>. As a result, they reject the notion of properties as universals. Properties exist in their ontology only as particular property-instances<sup>18</sup>, which in the literature are known as *tropes* but Heil prefers to call them *modes*<sup>19</sup>. Each object can have indefinitely many modes, but each mode uniquely characterizes the particular object which has the mode. In their ontology a property is only supposed to characterize, and the unifying role, which is usually understood as the job of a universal (they cite Kim<sup>20</sup> as an example of a view like this), is supposed to be performed by what they call the *types*, which are not properties but are *resemblance classes*. If two objects are of the *same type*, then they both are supposed to have properties belonging to the same *resemblance class*. Similarly, the more traditional way is to view a property as either categorical or dispositional. In fact, some philosophers<sup>21</sup> have envisaged the categorical properties as the lower level properties by virtue of which the dispositional properties as higher level properties can manifest themselves. Following C.B. Martin, however, Heil and Robb consider each property as *both* categorical and dispositional, just regarded from a different aspect<sup>22</sup>. When we put all of these above claims together, they claim that the result is a *no-layers*, lean ontology.

In their view, the layered view of properties stems from a confusion between properties and predicates. They claim that treating predicates as the same or similar to properties exhibits a misplaced faith in the relation predicates (language) have to properties (reality). They claim that under the influence of this wrong notion philosophers such as Block, Fodor and Putnam have allowed their arguments in philosophy of mind to shift from a claim about higher-level predicates to a claim about higher-level properties<sup>23</sup>.

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<sup>16</sup> See for a discussion on “sparse” or minimal notion of property: Swoyer, Chris. “Theories of Properties: From plenitude to paucity.” *Philosophical Perspectives*, 10, Metaphysics. Ed. James E Tomberlin. (Cambridge, Mass: Blackwell, 1996). 243-44.

<sup>17</sup> Heil and Robb, 186.

<sup>18</sup> *ibid.*

<sup>19</sup> Heil (2003), 12.

<sup>20</sup> Heil and Robb (2003), 178.

<sup>21</sup> Prior, E.W, Pargetter, R, and Jackson, F. “Three theses about dispositions”. *American Philosophical Quarterly*, 19 (1982.): 251-257.

<sup>22</sup> Heil and Robb (2003).

<sup>23</sup> Heil and Robb (2003), 177, 181.

Also, in their opinion the layered view of properties is at the root of a broad range of currently contested philosophical problems. For example, according to them, if one accepts that there are properties existing in their respective higher and lower levels, then the question rises how and whether the levels are connected causally. They see the controversy with *qualia* also as a problem of levels: it is either seen as a categorical property from one level, or as dispositional property from another level, thus its explanation is never wholly satisfactory to all sides. The solution is, they argue, to discard the multilevel ontology along with its metaphysical assumptions.

### 3. *What does it all mean for the mental properties?*

What does this mean for the mental properties? Does this new ‘sparse’, no-layer ontology have no room for the mental properties? Heil and Robb (2003) claim that it does. They say the mental properties are accepted in their ontology as “perfectly real”<sup>24</sup> complex properties, but not as “ontological additions”. Complex properties are supposed to be just elemental properties standing in a certain relation to each other. The creation of a statue, to use Heil’s example<sup>25</sup>, may require a certain complex arrangement of basic particles. From this, it *need not follow* that the universe contains statues *in addition* to the basic particles of physics, because, Heil contends, the *truth-maker* is the same. Every seemingly true statement that is affirmed about the statue, Heil claims, will have some complex arrangement of the basic particles as its *truth-maker*. Yet, he declines to be an eliminativist who claim that there are no statues. For, he argues that a statement such as ‘there are statues’ will be true because its truth-maker will be there<sup>26</sup>.

In other words, in this ontology a mental property is not an “ontological addition”. It is just a complex arrangement of basic physical properties. So, it does not exist “over and above” the physical properties at a higher and irreducible level. At the same time, predicates such as ‘is a belief’ will be meaningful without naming a corresponding property, and statements such as ‘there are beliefs’ or ‘there are desires’ will be true because the *same* truth-makers which would make statements such as ‘there are brain states....’ true will be there.

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<sup>24</sup> Heil (2003), 143.

<sup>25</sup> Heil (2003), 53

<sup>26</sup> Heil, (2003), 189.

Heil and Robb (2003) also mention that their view on the mental properties is compatible to the type-dualist views<sup>27</sup> and that the ‘mental types’,

...are not ontological additions to our world, they are simply more *abstract ways of characterizing* physical properties<sup>28</sup>. (italics mine)

As far as I understand, *ways of characterizations*, as mentioned the quotation cited immediately above, are predicates or descriptions; they do not carve up the reality. They belong to language, exhibiting our linguistic choice for this kind or that kind of expressions. If so, then the type-dualism that Heil and Robb want to endorse regarding the mental cannot be anything more than predicate dualism. In their ontology, then the mental ‘property’ is not really a property, as it is not considered really as “the way an object is”. Instead, it is admitted as one of the ways in which we may choose to describe bits of reality.

Finally, Heil and Robb claim that we may specify a situation in different ways as *Gus is in pain* (expression involving ‘mental’ term) or as *Gus is in brain state B* (expression involving only physical terms), but their truth-maker will be the same; namely, “Gus’s possession of one and the same property”<sup>29</sup>. This shows, they contend, that for the type of expressions containing the mental (a) we do not need to introduce any separate property layers, and (b) we also do not need to introduce separate or distinct property. In their own words:

...these various modes of specification *do not require, for their deployment, in re property layers*. ...such descriptions, while they classify the property differently, *do not introduce distinct properties*.<sup>30</sup> (italics mine)

#### 4. *Not a distinct property or not a higher level property?*

Is the claim:

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<sup>27</sup> Op.cit, 188.

<sup>28</sup> Heil and Robb (2003), 188.

<sup>29</sup> *ibid.*

<sup>30</sup> Heil and Robb (2003), 188-189.

(a) We do not need to introduce any separate property layers

equivalent to the claim:

(b) We do not need to introduce a separate or distinct property?

It does not seem so. (a) is an off-shoot from the general rejection of layered conception of properties. (b), on the other hand, is a far more stronger assertion. It is the negation of the irreducibility claim (2 or 2' of Section 1). Admission of (b) throws a serious challenge to all anti-reductionist positions. Moreover, one can agree to (a) without necessarily agreeing to (b). (a), if true, shows that the physical and the mental as properties do not need to be on two separate layers, lower and higher. But strictly speaking, that does not rule out the possibility of the mental being a distinct property at the same level. Elimination of layers in reality by itself does not establish that every property-instance, if it belongs to the same layer, must be the *same* or must be of the *same type*.

If they are not equivalent, then they should not be treated so. We need to evaluate separately which of (a) and (b) follows from what Heil and Robb (2003) state about the mental property. While doing so, we need also to remember what it is that we are evaluating. The question that we are concerned here is not the general question of whether it is possible to have an ontology without the mental as a property. That claim has been voiced by different groups of reductionists, physicalists, materialists, for years. Our task is to determine whether (b) *in this case* is supported by the arguments provided by Heil and Robb or not. Do their arguments show:

(a') the mental properties need not exist as higher level properties ?

Or, do they show that:

(b') the mental properties need not exist as distinct properties?

In order to close in on this, a good place to start is a direct quotation from them. Let me use a previously cited quotation from them again:

...these various modes of specification *do not require, for their deployment, in re property layers*. ...such descriptions, while they

classify the property differently, *do not introduce distinct properties*.<sup>31</sup> (italics mine)

Each of their arguments for the alleged misplaced faith in property-predicate correspondence, against universals, the supposedly indistinguishable nature of categorical and dispositional properties seem to lend support for the conclusion (a’): that an ontology can do without positing additional, hierarchical layers of being within reality. In their ontology, the connection between objects, as “property-bearers” or “propertied entities”, and properties is envisioned to be inseparable<sup>32</sup>, so this ontology does not allow different levels of being between objects and properties. They do not allow transcendental or immanent universals, hence for them there is no need for a higher ontological layer or plane to house the universals. They do not admit any level difference between categorical and dispositional properties either. Since there are no higher ontological levels, it follows that there are no higher-level properties also which “depend on, but are not reducible to, lower levels”<sup>33</sup>. If there are no higher level properties, clearly the mental property cannot be one of them. So, there is support in their arguments for the conclusion (a’).

But I do not see how the same arguments can also show that (b’): that the mental properties need not be properties distinct from the basic physical properties. Heil and Robb state that “the mental properties *are* ...physical properties”<sup>34</sup> follows as a consequence of their metaphysical position is. From which premises? As mentioned above, Heil and Robb try to answer this through an argument invoking parsimony: having the *same truth-makers*. In Heil (2003) Heil states it somewhat differently. He claims that if complex properties, even when they are “perfectly real”, are allowed to exist additionally as properties, then “sparseness evaporates”<sup>35</sup>. Let us look closely at both of these.

What exactly does having the *same truth-maker* show? A truth-maker is supposed to be a fact or a state of affair, or “some portion of reality”<sup>36</sup>, which makes a true statement about it true. Armstrong sees it as a cross-

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<sup>31</sup> *ibid.*

<sup>32</sup> Heil (2003), 172.

<sup>33</sup> Heil (2003), 7.

<sup>34</sup> Heil and Robb (2003), 188.

<sup>35</sup> Heil (2003), 143.

<sup>36</sup> Armstrong (2004), 5.



categorial relation, in which one of the relata is a truth or a proposition, and the other is some entity or item in the world<sup>37</sup>. There is no restriction on what a truth-maker has to be in order to make a true statement true: it can be whatever it takes to make the statement true.

Heil in his example of a statue combines with this truth-making a claim about metaphysical parsimony. Some dynamic arrangement of basic particles is supposed to be the *same truth-maker* for every possible true statement about the statue. This I understand as the claim that there will be a core set of properties which being basic will suffice as the common, shared truth-makers for *every* true statement about the statue. Similarly, we are supposed to assume that *every* true assertions or specifications about the mental will be made true by a common core set which will also act as *the same truth-makers* for the true statements about the physical.

But the mere fact that many true statements may share a group of properties as the *same truth-makers* does not by itself warrant the conclusion that other properties need not exist. For example, it is trivially true that every truth about this world has the world as the least common or the maximal truth-maker. From this, it does not follow that the existence of other properties as truth-makers is redundant. Even if we treat the claim of Heil and Robb of having the *same truth-makers* as having the same common *minimal truth-makers*<sup>38</sup>, even then the conclusion that they are after does not strictly follow. For, in a broad sense, some overlapping set of particles and their arrangement can certainly suffice as the same common minimal truth-makers for different truths. What makes it true that ‘My hair exists’ basically and minimally also makes ‘I as a human being exist’ true. From that fact, it does not follow that we need not introduce any distinct property which makes any of these distinct truths *individually* or *separately true*. Parsimony is not a blind metaphysical tool.

If we recall their discussion about the expressions ‘Gus is in pain’ and ‘Gus is in brain state B’, we shall find that Heil and Robb assume that both of these expressions can be made true by “one and the same property”. One

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<sup>37</sup> Ibid.

<sup>38</sup> “If T is a minimal truthmaker for *p*, then you cannot substract anything from T and the remainder will still be a truth-maker for *p*”, Armstrong (2004), 19-20.

might say that their claim may be interpreted as, not about maximal or common minimal, but about unique minimal truth-makers: both expressions have some property as identical *unique minimal truth-maker*. For a true assertion, a *unique minimal truth-maker* is supposed to be *one and exactly one*. However, this line of reasoning seems doubtful to me. For, all we know, there exists some property  $p$  that makes ‘Gus is in brain state  $B$ ’ true, and there exists some property  $p'$  which makes ‘Gus is in pain’ true. What ensures that  $p$  and  $p'$  are “one and the same”, i.e., not two? For, the ontology of properties of Heil and Robb is an ontology of modes or tropes. And an ontology of modes (tropes) allows the possibility of two exactly similar, yet numerically distinct, particulars. In this respect, tropes or modes, as particular entities, are said to defy the principle of identity of indiscernibles<sup>39</sup>. For the sake of argument, let us suppose that the modes of Heil and Robb follow this notion of particularity: that they can be exactly similar qualitatively yet be numerically distinct. Given this, in their ontology there is no *non-circular way* to establish that  $p$  and  $p'$  will be “one and the same”. Unless we *presume* already that the mental properties need not be among the properties, the claim about the *same truth-makers* do not rule out the possibility that we may still need the mental as a distinct property.

Can the appeal to “sparse” conception of properties preferred by Heil and Robb be used as a handy criterion to eliminate the mental property as a distinct property? Not, according to some. Schaffer<sup>40</sup>, for example, has argued that a “sparse” conception of property does not and need not exclude the mental property as a distinct property. Rather, he maintains that a “sparse” conception can be revised and redefined to include all such properties which, even if they do not belong to the micro-level fundamental physics, need to be invoked for a *total* “scientific understanding” of the world. This, on his view, includes the properties of mind as ontologically at par with the properties of the molecules.

Moreover, sparseness cannot be the ultimate guiding metaphysical criterion for allowing entities in this ontology of modes. As Heil himself notes<sup>41</sup>, the number of modes or particular property-instances or ways that each object is, will always exceed the number of objects in this ontology.

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<sup>39</sup> Williams, D.C. “Universals and Existents”. *Australasian Journal of Philosophy*, Vol. 64, No. 1 (1986): 3.

<sup>40</sup> See for example Schaffer (2004), Chalmers (1996).

<sup>41</sup> Heil (2003), 142.

This ontology admits unifying notions such as *types of modes* and some might argue that the typification of modes in this case may serve as a move to economize or to manage the plenitude of modes. However, we need to remind ourselves that in this ontology types do not occupy any ontological space. They merely help to classify the modes. Thus, bringing the modes or tropes under them does not really help to empty some ontological space. They merely classify the modes without helping to decrease the number of modes.

#### 4. *What does it all mean for antireductionist positions?*

Heil, and Robb, have brought to our attention the need to do metaphysics before doing philosophy of mind or cognitive science. They have also indicated that upholding a certain conception of properties can help us to do without a *layered conception of mental properties* that may be prevalent in the literature. Does this show that doing metaphysics of properties can necessarily lead a philosopher of mind only to the kind of weak predicate dualism that Heil and Robb endorse? Does this show that property dualism or any other anti-reductionist theory, which considers the mental property as a property i.e. as an ontologically irreducible item, is necessarily false or is mistaken?

I do not think so. It is a mistake to assume that the irreducibility claim of the mental (2 or 2' of Section 1) is a *consequence* or a *conclusion* that can be arrived *only* on the back of a claim of the mental property being a higher level property (3 of Section 1). The *irreducibility claim* of the mental property need not be considered as inalienably tied up with, or ensuing as a conclusion only from, a layered view of properties. It is possible to separate the irreducibility claim both analytically and ontologically from a layered view of the properties. That is, within an anti-reductionist position it is possible to defend 2 (or 2'), i.e. mental properties exist, without necessarily subscribing to 3, i.e. higher levels of properties exist and mental property is one of them. Just because A and B, two properties, are not reducible to each other in some sense, does not mean strata of being must separate them, or that any one of them has to be *higher* or *lower* in the order of being than the other. One may try the theory of *higher* and *lower* levels as a way to defend the irreducibility of the mental, but it need not be the only way to do so. Also, in the preceding section, I have argued that the ontology of Heil and Robb may be successful to persuade us to give up a view

of the world as containing stacked up hierarchies of properties and objects, with levels of being, but it does not satisfactorily show that the mental property is not needed as a distinct property.

Literature shows that the notion of irreducibility of the mental has received different treatment in the hands of different proponents. Where *p-types* represent the physical property group and the *m-types* represent the mental property group, given below are some of the different interpretations that the irreducibility claim of the mental property has received. These are not equivalent claims. If nothing else, they at least show that it is possible to conceptualize the irreducibility of the mental property in more than one way. Among these, only (g) explicitly appeals to different orders of reality:

- (a) *m- types* are not causally dependent on the *physical states* such as *brain states* or neurobiological states of the body<sup>42</sup>.
- (b) *m-types* can not ever be explained solely in terms of the concepts of the physical sciences<sup>43</sup>
- (c) *m- types* are not ontologically dependent on the *p-types*<sup>44</sup>
- (d) *m-types* are not logical consequences of the *p-types*<sup>45</sup>
- (e) *m-types* and *p-types* are not occupants of the identical functional role.
- (f) *m-types* are not just organizational features of physical matter<sup>46</sup>

To these, we can add also:

- (g) *m-types*, as a higher level property, cannot be given a complete and a satisfactory explanation in terms of the lower level physical properties.

In addition, there are other possibilities. Searle suggests a promising alternative. In Searle's metaphysics, the physical and the mental properties do not occupy two orders of being. For the sake of explanation, he allows consciousness to be understood as a high level system feature, but that does not mean it exists over and above the physical states and their properties. Yet, the mental is viewed to retain its distinction as an ontologically

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<sup>42</sup> Searle, John, R. "Why I am not a property dualist". *Journal of Consciousness Studies*, 9, No.12 (2002): 57-64. This is how Searle 2002 interprets property dualism, but Searle does not consider himself a property dualist.

<sup>43</sup> Churchland 1993, 10.

<sup>44</sup> Kripke 1997.

<sup>45</sup> Chalmers, 1996

<sup>46</sup> Churchland, 1993, 12.

irreducible property from the unique way in which we experience it. In his own words:

But in the case of consciousness, causal reducibility does not lead to ontological reducibility. From the fact that consciousness is entirely accounted for causally by neuron firings, for example, it does not follow that consciousness is nothing but neuron firings. Why not? What is the difference between consciousness and other phenomena that undergo an ontological reduction on the basis of a causal reduction, phenomena such as colour and solidity? The difference is that consciousness has a first-person ontology; that is, it only exists as experienced by some human or animal, and therefore, it cannot be reduced to something that exists independently of experiences<sup>47</sup>.

Sure, an anti-reductionist has a responsibility to metaphysically ground her claim of irreducibility. But, she has many choices. She may opt for Searle's understanding of ontological irreducibility to construct the rest of the theory of a mental property. Or, she may take the irreducibility of the mental property as a primitive notion or a brute. That is, it can be taken as a notion that is not further analyzable in terms of any further characteristics of the mental or that of the physical. This alternative does not preclude further theorization, as Heil and Robb state, "every theory must take some notions as primitive"<sup>48</sup> and their theory takes the similarity among the properties as primitive. Rather, it becomes the bulwark from which then a property dualist can build the rest of the account. This premise of distinctness between the mental and the physical as a given in the theory may create a metaphysical distance between the two, but it does not need to involve two separate orders of being. But it certainly does not warrant envisioning them as two separate realms with no bridge in between. In any case, she does not have to embrace the layered ontology of properties that Heil and Robb find so unacceptable.

To conclude, rejection of a layered ontology does not show that the basic irreducibility claim of antireductionist theories such as property dualism must also be forsaken. This is why, contrary to what Heil and Robb claim, dismissal of higher and lower property layers does not effectively solve the "myriad philosophical puzzles" in philosophy of mind. They may wrongly suppose that all of them come from conceiving the mental property as a higher-level property, when actually many of the problems stem from a ba-

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<sup>47</sup> Searle, 2002, 60.

<sup>48</sup> Heil and Robb, 184.

sic claim of irreducibility of the mental. For example, they claim that the problem of *causal relevance* of the mental will go away, once we accept their zero-level metaphysics. Since in their ontology there will be no property layers, higher and lower, competing for causal relevance, so they argue that there will be no question about whether and how the mental can be causally relevant for behavior. But the problem of causal relevance poses a different question that starts from the irreducibility claim: How can the mental be causally relevant to our behavior in a causally closed universe, *given that the mental exists*? It is a problem *only if* one's metaphysics is anti-reductionist. The "solution" that Heil and Robb offer does not address the irreducibility claim that is built into the premise of the problem and merely dismisses the problem as a non-issue once the metaphysical levels are collapsed.

AMIHUD GILEAD

## **A Possibilist Metaphysical Reconsideration of the Identity of Indiscernibles and Free Will**

ABSTRACT.

I. If we consider any two entities, such as the two spheres in Max Black's thought-experiment, as possibilities, pure or actual, they cannot be considered indiscernible at all. Since allegedly indiscernible possibilities are necessarily one and the same possibility, any numerically distinct (at least two) possibilities must be discernible, independently of their properties, "monadic" or relational. Hence, any distinct possibility is also discernible. Metaphysically-ontologically, the identity of indiscernibles as possibilities is thus necessary, however epistemic discernibility is still lacking or does not exist. Since any actuality is of a single pure possibility, the identity also holds for actual indiscernibles. The metaphysical or ontological necessity of the identity of indiscernibles renders, I believe, any opposition to it entirely groundless.

II. Like pain, the experience or feeling of free will is subjective yet infallible and authoritative from intersubjective or objective perspective as well. Whether the grounds for being in pain are known or not, being in pain is infallible. The same holds for our experience of free will. As much as no illusion of pain is possible, no experience of free will is possibly an illusion. As much as the experience of pain constitutes the reality of pain, the experience of free will constitutes its reality. In both cases *percipi* is *esse*. The freedom of will is thus immune against illusion or self-deception, whether the will is motivated or not, determined or not, and whether the reasons or causes for its determinacy or indeterminacy are known or not. The unintelligibility or the mystery of free will does not cast any doubt on its reality as a well-established fact.

### *(I) The Identity of Indiscernibles Reconsidered*

The principle of the identity of indiscernibles has been supported and also strongly attacked.<sup>1</sup> Max Black's attack (1952) on it deserves special attention.<sup>2</sup> As I will show below, the identity of indiscernibles can

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<sup>1</sup> Leibniz, Russell, Whitehead, F. H. Bradley, and McTaggart supported it, whereas Wittgenstein (the *locus classicus* is *Tractatus* 5.5302, criticizing Russell and arguing that two distinct objects may have all their properties in common), C. S. Peirce, G. E. Moore, C. D. Broad, and Max Black are among its strong opponents. The support may adopt an idealistic stance, while the opposition is clearly anti-idealistic or empiricist.

be secured on a metaphysical basis regardless of any form of the principle of sufficient reason or any other Leibnizian consideration.

Black suggests the following counter-example to the identity of indiscernibles:

Isn't it logically possible that the universe should have contained nothing but two exactly similar spheres? ... every quality and relational characteristic of the one would also be a property of the other. Now if what I am describing is logically possible, it is not impossible for two things to have all their properties in common. This seems to me to *refute* the Principle. (ibid., p. 156)

This counter-example consists of a possible world (“universe”) in which no observer is present and exact duplicates, exactly similar objects, identical twins, and the like, all of which are indiscernible but not identical, may exist (ibid., pp. 160-62). I will show why on metaphysical-possibilist grounds no such possible world could exist.<sup>3</sup> Thus, independently of the question of common properties, relational or not, of bundles of properties as universals, or of “predicative functions” (the term that Russell and Whitehead’s theory of types employs), I will show why indiscernibles (or indistinguishables) that are not identical are metaphysically impossible. Even if Black’s aforementioned possible world is logically possible, it is nonetheless metaphysically or ontologically impossible.

Let us begin with the definitions of some terms that I will use in this paper. Regardless or independently of any actuality or actualization, all possibilities are pure. By “possibilities” I have no possible worlds in mind but individual possibilities (or possible individuals) instead. My possibilist stance is entirely independent of any conception or semantics of possible worlds. Possibilism is an ontological or metaphysical view according to which pure possibilities do exist. In contrast, actualism is the view that only actualities exist, and possibilities are merely the ways in which such

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<sup>2</sup> Black’s arguments have been discussed by Hacking (1975), Adams (1979), Casullo (1982), Denkel (1991), Landini and Foster (1991), French (1995), Cross (1995), O’Leary-Hawthorne (1995), Vallicella (1997), Zimmerman (1998), and Rodriguez-Pereyra (2004). Nevertheless, there is still room enough for alternative treatments of it on quite different grounds (especially different from those of fictionalism, the bundle theory, or haecceitism).

<sup>3</sup> The possibilist metaphysics to which I refer in this paper is entitled “panenmentalism.” I introduced it in Gilead, 1999 and elaborated it in Gilead, 2003.



actualities might have existed. Possible worlds have been considered among such ways. Hence, actualism is compatible with some conceptions of possible worlds but not with any ontological standing of pure possibilities (possibilities *de re*). When we apply “existence” to pure possibilities, the term serves us in a non-actualist sense. Since pure possibilities are individuals and not universals or bundles of universals, there are no instances of them. Against many current views (such as Rescher’s 1999 and 2003), we are capable of identifying and quantifying or enumerating individual pure possibilities (Williamson 1998, 1999, and 2000, discussing individual “mere” or “bare” possibilities; Gilead, 2004b). Furthermore, we can rely upon individual pure possibilities as the identities of actualities. If each actuality is an actualization of an individual pure possibility and of no other possibility, the pure possibility serves as the identity of the actuality in question. As pure, such possibility-identity is not spatiotemporally or causally conditioned, whereas any actuality is inescapably so conditioned. Actualities are accessible by empirical means, whereas pure possibilities—logical, mathematical, metaphysical, or otherwise—are accessible to our thinking and imagination. As thus accessible, pure possibilities are discoverable as much as actualities are (think of the discoveries of mathematical or logical possibilities, which are not empirical at all), but this must remain beyond the present paper (see Gilead, 2004b). As I will argue below, when it comes to individual possibilities, any distinction makes a qualitative difference.

To return to Black’s thought-experiment, first we need a criterion of identification to denote or name something. To defend the principle of the identity of indiscernibles, I assume a criterion of identification of pure possibilities that does not rely upon relational properties and spatiotemporal distinctions. Were such properties and distinctions inescapably required to establish the principle, Black’s view would have appeared to be more sound. Is Black right in stating that mere thinking is not enough to identify or name a thing (*ibid.*, p. 157)? Black assumes that to identify or name anything we need a denotation of an actual object or a unique description of it (*ibid.*). Such need not be the case at all. Think, for instance, of eka-elements in the periodic table. Each such element is not actual but is a predicted pure possibility (Gilead, 2003, pp. 65-70). Many mathematical theories, let alone all the pure possibilities which they comprise, were discovered only by creative thinking or imagination, while identifying, naming, and describing any of these possibilities have been

quite practical with no recourse to actualities. Indeed, to discover, refer to, identify, or name pure possibilities, thinking or imagination is more than enough. We are certainly capable of denoting pure possibilities, each of which is uniquely describable, for, as I will argue below, no two pure possibilities can be indiscernible. Second, pure possibilities-identities are necessary for identifying, denoting, searching for, detecting, and describing the relevant actualities, although we also need empirical means to do so.

There are two ways to interpret Black's thought-experiment, which is a counter-example to the principle of the identity of indiscernibles. First, the two spheres are merely pure possibilities.<sup>4</sup> Second, the two spheres are actualities. In the second case, they must be subject to spatiotemporal and causal conditions, as no actuality is exempt from them. In the first case, they are exempt from such conditions altogether, for no pure possibility can be subjected to them. In both cases, the spheres are possible, for any actual thing is possible too. This means that in both cases we have two possible spheres with the following difference: in the first case, the possibilities in question are pure, whereas in the second—they are actual.

What is precisely the distinction between *b* as a pure possibility and *b* as an actual possibility? The pure possibility in question comprises all the pure possibilities that are open to *b* under one and the same identity, whereas *b* as an actual possibility comprises only some of them, namely, only those that have been actualized. The actualization of any of these possibilities does not change the pure possibility-identity of *b*, which is one and the same possibility despite any change that *b* as an actuality may undergo. For instance, James Joyce could have not written *Finnegans Wake* and yet he would have been the same James Joyce under one and the same pure possibility-identity (namely, the only possible author of *Dubliners*, *Ulysses*, *Finnegan Wake*, or other masterpieces). Note that *b* as an actual possibility and *b* as an actuality are one and the same *b*, both comprised in one and the same pure possibility-identity. All these distinctions are within one and the same pure possibility-identity, which

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<sup>4</sup> Pure possibilities are exempt from any spatiotemporality. Can a sphere as a pure possibility be exempt from space? Yes, it can. Think of any figure, such as sphere, in the analytical geometry, which transforms any spatial distinction to algebraic properties. In Kantian terms, even algebraic properties are subject to temporality, since the arithmetic series is subject to it. But my view is by no means Kantian, especially concerning spatiotemporality and the identity of indiscernibles. As a result, as pure possibilities, the two spheres are entirely exempt from spatiotemporality.

does not render it into separate individuals. In other words, *b* as actual and changeable or *b* as an actual possibility, which is neither changeable nor spatiotemporally and causally conditioned, takes part in one and the same pure possibility-identity. As actual, *b* is the spatiotemporally and causally conditioned part of *b* as a pure possibility. No actual individual exhausts all the possibilities that are open to it; it might always have been actually different and yet necessarily remaining one and the same individual under (“comprised in”) one and the same pure identity-possibility. This possibilism *de re* requires no transworld identity, possible worlds, possible counterparts, or any haecceity (qualitative or nonqualitative “thisness,” such as Adams’s), each of which appears to give rise to further problems and vagueness instead of providing us with some clear answers.

No two pure possibilities might be indiscernible and yet not identical. Independently of any properties, “monadic” or relational, any allegedly “two” indiscernible pure possibilities, discoverable by means of our imagination or thinking, are indeed one and the same possibility. To think about or to imagine two pure possibilities necessarily means to distinguish between them, to discern the one from the other, with no recourse to spatiotemporal distinctions at all. Any pure possibility is exempt from any spatiotemporal or causal conditions. Hence, no pure possibility is spatiotemporally located. If, nevertheless, there are really two of them, they are distinct because they are qualitatively different, not because they are in different places at the same time. They relate one *to* the other because they are different one *from* the other, not the other way round. Since any actuality is of a single pure possibility-identity, necessarily, according to such metaphysics, no indiscernible yet non-identical pure or actual possibilities exist.

Could any actualist counter argue that s/he had not the slightest idea of how could one have any access to the pure possibilities-identities of the two exactly similar spheres in one of the above possible interpretations of Black’s thought-experiment? No, for all we need is something like such a thought-experiment to have access to the pure possibilities-identities of these two spheres. Indeed, Black unknowingly “provides” these possibilities in his imaginary experiment or logically possible universe, which is not confined to the actual one. All we need is our imagination, within the domain of logical possibilities (as Black assumes on p. 156) or without it, to be acquainted with pure possibilities such as these two. Even

if no such spheres existed in our actual universe, Black could suggest his aforementioned thought-experiment because he, like any person who is endowed with imagination, has access to the realm of the purely possible. What makes such an experiment possible is simply our accessibility to that realm by means of our imagination, logic, mathematics, metaphysics, and other ways of thinking, all of which should not be confined to the actual. My interpretation that the two spheres can be either pure possibilities or actualities that actualized these pure possibilities holds true for Black's thought-experiment. Black would certainly agree that no two possibilities whatever can be identical, for "two" identical possibilities are really one and the same possibility.

The question is: are these two spheres, as pure or actual possibilities alike, not only non-identical but also indiscernible? Like "two" identical possibilities, "two" indiscernible possibilities are simply one and the same. There are not two of them at all. It is easier to realize that in the case of pure possibilities discernibility must be obvious. For in that case we have no recourse to actualities or to any of their conditions or terms. Can you think of, or imagine, two pure possibilities without discerning one from the other? No, since there are no two indiscernible pure possibilities. Indiscernibility of pure possibilities, if possible at all, would necessarily imply that there were no pure possibilities but only one. As far as pure possibilities are concerned, indiscernibility implies identity. If the two aforementioned spheres are pure possibilities, they must be discernible as well as not identical.

As we shall realize, the same holds true for the two spheres as actual possibilities. As far as actual possibilities are concerned, they too are necessarily discernible as well as not identical. Otherwise, the two spheres, as actual possibilities, would not have been considered two actual possibilities but only one.

Yet Black could answer back on another basis. He would restate his claim that there is no way of telling the spheres apart (*ibid.*, p. 156), which implies, to return to my view, that even if we have access enough to the pure possibilities-identities of the spheres, how can we ascribe possibility *b*, for instance, to one of the spheres, given that we are entirely incapable of telling the spheres apart? In other words, how can I identify one of the spheres as an actuality of possibility *b* rather than of possibility *c*? In this

case, my accessibility to the pure possibilities-identities of the spheres appears not to be helping me to identify any of the actual spheres. Which is which if there is no difference to tell? Yet this would not help Black at all. For the problem of identification or recognition of actualities is epistemological and empirical, not ontological-metaphysical. We have to distinguish between identity, which is ontic, and identification, which is epistemic. We have also to distinguish between identification of pure possibilities, which requires no empirical means, and that of actualities, which requires such means in addition to the identification of the relevant pure possibilities-identities. Suppose that I cannot know which actual sphere is which, I still know for sure that either sphere must be ontologically-metaphysically discernible, for each is an actuality of a different possibility-identity, whether I can tell the difference between the actual spheres or not.

If the spheres in question are actual, they must be different one from the other, for no two actualities can be of one and the same pure possibility-identity. Elsewhere I have shown that multiple actualization or “realization” of any pure possibility should be excluded (Gilead, 1999, pp. 10, 28; Gilead, 2003, p. 94). Apart from this, since any actuality is also a possibility (but not the other way round), and since any indiscernible or non-distinct possibilities are identical, and are one and the same possibility, any two—namely, at least numerically distinct—possibilities cannot be identical and are discernible on ontological-metaphysical grounds. The epistemological discernibility must follow the ontological-metaphysical discernibility of possibilities, pure or actual, not the other way round.

On the grounds of possibilities alone the identity of indiscernibles is metaphysically secured beyond any possible doubt. Even regardless of their properties, “predicative functions,” and relationality, absolutely, no two possibilities can be metaphysically indiscernible, otherwise they would have been merely one and the same possibility. Hence, with possibilities, pure or actual, numerical distinctness and qualitative difference are entirely compatible. No spatiotemporality, any other possible principle of individuation, or property is needed for the discernibility of any possibility. No two possibilities can be indiscernible, let alone identical, whatever are their properties, relational or not. The identity of each actuality is necessarily determined by its pure possibility-identity alone. No two actualities can share one and the same possibility-identity.

Note that my possibilist view does not acknowledge any spatiotemporal principle of individuation. All those classical empiricists or Kant (according to whom space and time are the forms of intuition or the only factors of individuation), who endorse spatiotemporal principle of individuation (*principium individuationis*) challenge the principle of the identity of indiscernibles in general or Leibniz's principle in particular. For they all assume the irreducibility of spatiotemporal differences to more fundamental or "primitive" factors of individuation. In this respect, Kant challenges that principle. According to him, like Locke, indiscernibles all of whose properties are common are not identical, for they exist in different places at the same time. Hence, this is sufficient to make indiscernibles numerically distinct. In contrast, my view, like Leibniz's, is that numerical distinctness of actualities indicates qualitative difference. Since actualities differ qualitatively, they are numerically different, not the other way round.

Black's possible world in which indiscernibles—duplicated particulars or worlds—are not identical is a narcissistic nightmare: "A kind of cosmic mirror producing real images... except that there wouldn't be any mirror" (ibid., p. 160). For a possible world in which "everything that happened at any place would be exactly duplicated at a place an equal distance on the opposite side of the center of symmetry" (ibid., p. 161) is a world in which no difference exists between an object and its mirror image. Suppose now that on epistemic grounds we cannot distinguish between two poles of a gravitational or magnetic field, two electrons, and the like (Black's examples on p. 162). If Black's possible world is a cosmic mirror, it is inferior to any world in which mirrors exist and in which we can distinguish between any object and its mirror image. Only due to some brain damage do adults become incapable of distinguishing between themselves and their mirror images or of recognizing such images as theirs. Notwithstanding, suppose that we know for sure that two things (two poles, two electrons, an object and its mirror image, and the like) exist in Black's possible world although there is no way to realize any difference between them, such indiscernibility carries no ontological commitment whatever. All we can say is that we do not detect any difference, which is an epistemological question, but we are absolutely not entitled to conclude that no such difference *exists* at all. Unlike Black's examples, in which the presence of an observer changes the possible universe (ibid., which follows

quantum mechanics), pure possibilities-identities are discoverable by us yet their existence and the differences they “make” or bear are entirely independent of our knowledge. Think again of eka-elements, mathematical pure possibilities, and the like; these were all discovered, not invented.

The two exactly similar or duplicated spheres that “exist” in Black’s possible world are not identical only because, contrary to his argument, they are discernible. For, first, if they are merely pure possibilities, they are necessarily discernible, as no two (“numerically distinct”) pure possibilities can be indiscernible. And, secondly, if the spheres are actual, either must be an actuality of a different pure possibility-identity, no matter what relations, spatiotemporal or otherwise, exist between the spheres or between any of them and any possible observer. Thus, contrary to Black’s view (*ibid.*, p. 163), there is always a way in which any thing, purely possible or actual, is different from any other. On these grounds, Black’s arguments should not convince the readers at all, contrary to the ending of the article (*ibid.*, p. 163), in which interlocutor *A* in Black’s imaginary dialogue declares himself not convinced by *B* (Black)’s argument, while *B* responds, “Well, then, you ought to be” (*ibid.*). This is an excellent example for an “overwhelming” argument, which *A* is unable to refute and which, yet, is entirely blind to an illuminating insight about the ontological-metaphysical necessity or indispensability of the identity of indiscernibles.<sup>5</sup> I strongly recommend following that insight, which may open one’s eyes to realize why that identity is a metaphysical necessity. In this paper I have attempted to support this insight with a possibilist argument.

But suppose that Black rejects any possibilist view. Suppose that he argues against me that pure possibilities are merely nonsense (or that they are only *de dicto*, never *de re*), that only actual things can exist, and that his possible world or thought-experiment is not about pure possibilities but about actualities in the very actual world in which we live. Nevertheless, I could answer him again that since any actual thing is possible too, and since two possibilities that no difference exists between them are merely one possibility, the identity of indiscernibles is well secured. In other

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<sup>5</sup> For some other instructive examples of blind arguments versus illuminating insights see Gilead, 2004a.

words, merely on modal grounds, actualist or otherwise, Black's view against the identity of indiscernibles holds no water. On the other hand, if he will not take modality seriously, and if the possible, pure or actual, implied no ontological commitment whatever, Black could defend his view at some unbearable cost, that is, rendering modality and especially possibility ontologically insignificant.

To attempt to persuade the actualist who does not accept any possibilist assumption or principle, the argument that the two spheres are actual possibilities should be good enough. If the term "pure possibilities-identities" do not make sense for actualists, they, nevertheless, must consider the two spheres either as actual possibilities or as the possible modes ("ways") in which the actual spheres might have existed. In either case, those spheres are possibilities too, and no two indiscernible *possibilities* that are not identical can make sense for actualist or possibilist metaphysicians alike.

Let us reconsider the case of two actual "indiscernible" spheres from the aspect of spatiotemporality. In Euclidean space the case appears to be to some opponents of the identity of discernibles, from Kant on, that indiscernibles are not identical, for, sharing all their qualities, they are still "spatially dispersed, spatially distant from one another" (Adams, 1979, p. 14), which makes them numerically distinct. Surely, as far as the space in Black's possible world is Euclidean, there are two spheres although no difference between them is discerned. Consider now these two actual spheres as actually possible, namely, as two actual *possibilities*. As possibilities, they are not spatially or temporally dispersed (at most they are spatially or temporally *dispersible*), for no possibility, pure or actual, is spatially or temporally locatable. As actually possible, the spheres are two, not because they are spatially or temporally dispersed but rather because they are two qualitatively different possibilities and, *hence*, numerically distinct. Temporally dispersed actualities (namely, events) must be first and foremost qualitatively different because their ontological grounds or "primitives"—their possibilities—are qualitatively different. The *possibility* of being spatially or temporally dispersed, which is not spatiotemporally conditioned, is metaphysically prior to any actual spatial or temporal dispersal. In the final account, the pure possibilities-identities, which are absolutely exempt from any spatiotemporality, are the metaphysical-ontological grounds of the qualitative difference as well as



the numerical distinctness of any individual actuality. In any case, were the two spheres not actually possible in the first place, they could not be two actual spheres spatially distant from one another. They would have been then one and the same sphere, namely, identical to itself. In this way too the identity of indiscernibles is *necessarily* maintained. Individual distinctness, such as numerical distinctness, is intelligible only dependently of qualitative difference (contrary to Adams, 1979, p. 17). Black's counterexample to the identity of indiscernibles is thus refuted even when actual spheres in Euclidean space are concerned.

As for a non-Euclidean space or curved time, it has already been shown that on the grounds of spatial or temporal dispersal two indiscernible actualities can be identical.<sup>6</sup> In such space or time, one and the same object may be spatially or temporally distant from itself. Yet, the point is not to show that the identity of indiscernibles is possible but rather that on metaphysical grounds it is necessary, to show that there is no possible single example in which indiscernibles are not identical. Bearing in mind my arguments so far, I have shown that there is no such example and that no such example can be found. As a result, the identity of indiscernibles is necessary, not only possible.

The apparent advantage of my possibilist treatment of the question of the identity of indiscernibles is, I think, that it equally holds for pure possibilities and actualities and, hence, clearly demonstrates that it is impossible for indiscernibles not to be identical. Both Leibniz's illustration of the discernibility of each leaf of an actual tree and, considering all the differences, C. S. Peirce's "no doubt, all things differ; but there is no logical necessity for it"<sup>7</sup> are aimed at actual things. What I have shown above is that there is a metaphysical or ontological necessity for the identity of indiscernibles, which, I believe, renders any opposition to it entirely groundless. For those who oppose this identity and who also assume that metaphysical and logical necessity are one and the same, the case appears that I have also proven that the identity of indiscernibles is logically necessary. In sum, my arguments, possibilist or otherwise, clearly

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<sup>6</sup> Consult Adams (1979, pp. 13-17), following Black (1952, p. 161) and Hacking (1975). Cf., however, Denkel (1991, pp. 214-15, footnote 3), Landini and Foster (1991, pp. 55-60), and French (1995, pp. 461-466).

<sup>7</sup> As quoted in Black (1952, p. 163); cf. Casullo (1982, p. 595-596), Landini and Foster (1991, pp. 54-55, 58-60).

show that the non-identity of indiscernibles is merely impossible, logically, ontologically, and metaphysically alike.

Finally, it is because any pure possibility is discernible from any other that the possibilities in question do not share all their properties, relational or otherwise, and not the other way round. Because any two pure possibilities are discernible, they must differ in their properties too. Because any two pure possibilities are necessarily distinct and different one *from* the other, they necessarily relate one *to* the other, not the other way round. Hence, Leibniz's principle of the identity of indiscernibles should be modified on that possibilist basis. Every thing must be distinct and different from any other thing, not just because they do not share all their properties, but primarily because their pure possibilities-identities necessarily differ one from the other. Because of this difference, they cannot also share all their properties.

(II) *Is Illusion of Free Will Possible at All?*

Not a few philosophers have been convinced that free will is merely an illusion (for a recent example consider Smilansky, 2000). The most notable is Spinoza, especially in the *Ethics*, according to which the fiction or illusion of free will is a result of ignorance or an error.<sup>8</sup> In this paper I will make a metaphysical comment challenging the possibility of such an illusion altogether and explaining why we should be ontologically committed to free will.

Some mental states—such as being in pain, feeling well or unwell, comfortable or uncomfortable, stressed or relieved, calm or agitated, and experiencing one's will as free or one's desire as compelled—cannot be illusions. To experience any of these states is what its reality is all about; all its *esse* is simply *percipi*. The experience alone is sufficient to constitute the state of one's mental, subjective reality. The reality that such experience constitutes is one and the same with the experience itself and it must

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<sup>8</sup> *Ethics* 1App, Spinoza, 1985, p. 440:17 ff.; *ibid.*, 2p35s, p. 473; 2p48 and s; 2p49s, pp. 484-491; 3p2s, pp. 496:13-497:30; and 4p1s, pp. 547-548. The first number in each reference refers to the number of the part; "App" designates appendix; "p" proposition; "s" scholium or note; "d" demonstration; "p." or "pp." stands for the pagination of Curley's translation, while the numbers after the colon designate the lines.

not rely upon anything else. Such is not the case of any illusion. Illusion is mentally, subjectively real as far as the mental state of the person under it is concerned, but it inescapably refers to something else that is not the experience in question. This involves two things: (1) the existence of the illusion and (2) its referred object (which may not exist). The illusion must be about something else, distinct from the illusion itself. In contrast, to experience one's will as free is a state of one's mental, subjective reality, and it does not refer to any other fact, mental or otherwise, existent or nonexistent. The experience and the fact are one and the same. The *percipi* of free will alone makes all there is about it, its *esse* as a whole. Equally, to experience any pain is *to be in* pain. The *percipi* of any pain alone is its *esse*. No other *esse*, alleged or real, must be involved with the experience of free will or with that of pain. To experience or feel one's will as free must not refer to something else but only to the experience itself. To experience it is not a reflection about something, as much as pain is not a reflection about something. Neither involves any introspection. Above all, no room is left for illusion or mistake about such mental states as such, for none of them is a belief or knowledge, which are fallible and may turn to be illusions. Finally, none of them is a representation of a mental state; it is rather the mental state itself. One's experience of free will does not represent free will as a mental fact; it is rather one's mental fact itself. Equally, one's experience of pain does not represent any pain as a mental fact; it is rather the reality of one's pain. To experience free will makes a mental reality of free will.

In contrast, quite different mental states, whose *esse* is not simply *percipi*, are both subjective and cognitive. To experience or have such states does not constitute any mental reality or fact to which the experience refers. Thus, each of such states is fallible and can be merely an illusion. If James believes himself to be omniscient or omnipotent, this does not make any fact about his real capability or about his mental reality to which this illusion refers and which is different from the illusion. If, actually, he is absolutely incapable of writing an excellent paper in philosophy, for instance, even though he considers himself capable of doing so, his belief is by no means sufficient to render him capable of achieving that. All the mental states of this kind are subjective, cognitive, and absolutely fallible. All of them refer to some mental fact or reality that is beyond them. In fact, each of the aforementioned examples is merely an illusion about one's mental state or capability. As such, they erroneously represent such state or capa-

bility to which they refer. The *percipi* of the illusion does not constitute the *esse* of the referent, of the mental capability or any other capability of the person under that illusion despite his or her strong belief, “knowledge,” conviction, or self-consideration. In contrast, the case of the experience or feeling of free will or of pain is entirely different. No fallibility has any room in any of such states. As I will argue below, their subjectivity bears intersubjective and objective veridical standing or truth, and it is absolutely impossible for any of them to result in illusion or self-deception.

Galen Strawson rightly rejects any possibility that pain is illusion or mere seeming, for “the seeming is itself and ineliminably a real thing” (1994, p. 51), and argues that to consider pain as illusion is simply irrational (*ibid.*, p. 53). Indeed, as he shows elsewhere (1986, pp. 222-225), the *esse* of pain is *percipi* or “pain just *is* pain-experience.” Nevertheless, for reasons that will be further explicated below, I do not see how such an understanding of pain is compatible with the assumption that “there is no such thing as free will” (*ibid.*, p. v).

No one, however capable or knowledgeable, is entitled to deny any of your pains. Such denials should be considered totally irrational or groundless. Furthermore, absolutely no one is entitled to argue that the pain in question is merely illusion. We are entitled to disbelieve or discredit one’s *complaints* or *claims* about one’s pain, since his or her behavior, reaction, appearance, and the like indicate, to our best judgement, that this person is not in pain. Nevertheless, no one is entitled to disavow the *reality* of pain or being in pain, even if its reflection on the relevant objective or intersubjective reality is not recognized. Even if an able physician finds no grounds for the patient’s complaint about pain, she is entirely incapable of denying the reality of that pain or of diagnosing it as a mere illusion. The patient may be in pain even if no external, objective or intersubjective, indications or grounds for it are recognized at all.

The reality, the very existence, of pains or other subjective states must not depend or supervene on objective-impersonal or intersubjective-interpersonal reality. Subjective experiences, such as being in pain, do not require any use of language, for language rests upon intersubjective reality. There is no private or objective language; only intersubjective languages exist. We need knowledge and language, both of which are intersubjective, to name, define, or describe our mental states; but to experience or realize

them no language or other intersubjective devices are needed. Infants are subject to pains, stress, pleasure, relief, and the like very early in their life, well before any command of language. Equally, we must not rely upon language, knowledge, belief, or any other intersubjective means to feel free, coerced, relieved, and the like. One's experience of free will requires no knowledge, belief, or language.

What is it like to experience or feel free will? Whenever, under no compulsion or force, I follow my volition, I feel or experience the freedom of my will, entirely exempt from any coercion or constraint. I feel "like it," I *freely* want it as it is, and I fully ("integratively") stand by my will. Under compulsive or addictive desires, no one can feel one's will as free. One feels whether or not any coercion, compulsion, or addiction is involved in whatsoever way with one's volition, and one can certainly distinguish between free will and coerced or compelled desire. To experience or feel free will does *not* mean to have or consider it unmotivated, undetermined, or uncaused. Having free will is entirely compatible with being determined or motivated, whereas coercion or compulsion is incompatible with free will. I will return to this point below.

Under hypnosis, patients may experience their will as free. Could this serve as a counterexample to the argument that the experience of free will must be exempt from any illusion? No, for hypnosis consists of *self-suggestion* in which the patients help themselves to be exempt from their inhibitions, to be relieved from some constraints. In fact, the patient's self-suggestion mobilizes or utilizes the aid of the hypnotist to get such a desirable affect. No one can be hypnotized against one's free will. Experience or feeling of free will under hypnosis is as real as in normal life except for the capability of hypnosis or self-suggestion to relieve the patients from some inhibitions that constrain their experience of free will. This experience in itself cannot be unconscious just as no unconscious pain exists. Hence, inhibitions or constraints may eradicate or suppress, not repress, one's experience of free will. In conclusion, under hypnosis too, the patients' experience of free will, like the patients' experience of pain, cannot be illusion.

What about unconscious grounds which if one was conscious of, one would have not felt one's will as free? In such case, is not one under an illusion of free will? As far as effectiveness is concerned, there is no differ-

ence between conscious and unconscious grounds. In either case, the effect, namely, experiencing one's will as either free or coerced, must be conscious or felt. Consequently, if one feels one's will as free, no grounds, conscious or unconscious, exist for him to feel otherwise. The same holds for one's feeling oneself under coerced desire. Hence, rendering any unconscious grounds conscious, would not change even slightly one's feeling of free will or that of being under compelled desire. In conclusion, whenever one feels one's will as free, no illusion about it due to unconscious grounds can take place.

To feel exempt from any coercion or addiction is as infallible as being exempt from any pain or being in pain. Everybody can simply recognize the infallible distinction between being in pain and being exempt from any pain, of being coerced and of being exempt from any coercion, of having free will. One is certainly capable of *taking* one's will as free, whereas no one is capable of *mistaking* one's will as free, just as one cannot mistake oneself as being in pain or as being exempt or relieved from any pain. Whenever you feel yourselves as having free will, there is absolutely no mistake or doubt about it.

Nevertheless, I may be mistaken about some of my emotions and misidentify them. For instance, I may feel angry about something or somebody, although what I really, truly have "deep down" is quite another emotion, say, fear or jealousy. To recognize that, my experience is not sufficient and introspection as well as knowledge or other intersubjective means are required. Similarly, could I have a strong sense of free will although "deep down" I might unconsciously have something very different? Could not my sense of inner freedom be then merely an illusion? Indeed, fear, jealousy, and the like may *appear or be experienced* as anger, and in a sense I may be under the illusion or self-deception of being angry. Unlike being in pain or having free will, emotions can be unconscious (Gilead, 2003, pp. 160-162), and the *percipi* of any emotion can be different from its *esse*. Hence, we may be wrong about the unconscious emotions behind our feelings but not about the feelings or experiences themselves, all of which are conscious. Having free will cannot appear, be experienced, or felt as a different mental state, just as pains cannot appear or be experienced as other feelings or sensations, for the *esse* of pain or of free will is *percipi*. Consequently, unlike some of my emotions, I could not misidentify or be mistaken about my free will.

While in pain, you are incapable of mistaking your pains for other pains, sensations, or feelings, and certainly you are not self-deceived or under illusion. Some of your “physical” pains may have no physical grounds whatsoever, and an expert may suggest that you experience distress or some mental stress as if it were a physical pain, although no physical grounds for this pain exist. Nevertheless, you undeniably experience “physical” pains then (since any pain is mental, I use “physical” qualifiedly), and there is no illusion about that experience. No painful situation is an illusion or mere appearance (whereas being angry may be merely the appearance or experience of another emotion); its *esse* is *percipi*. You are capable of mistaking or misidentifying the significance or the causes of your pain, not its nature or identity. Such is also the case of phantom pains, which are unmistakably pains. Being in phantom pain, a person believes it to indicate or signify some occurrence in a nonexistent, amputated limb. Nevertheless, the pain as such involves no illusion; only the belief or judgment as to the origin, causes, or significance of the pain is fallible. No one, however omniscient or omnipotent, can challenge the reality of one’s pain, phantom or not.

Equally real is the infallible experience that some of our volitions are entirely free or that our will is free in such cases. As Richard Griffith puts it, we “cannot do away with the compelling reality of the experience of free will” (1962, p. 232, nevertheless, we should do away with both his “as-if” and “no metaphysics” concerning free will). However motivated, determined, conditioned, manipulated, or coerced persons may be, their feeling or experience of free will should be unquestionable, no matter to what extent they are hetero-determined or self-determined. However compelling, forcing, or constraining the circumstances under which they are acting, feeling, or thinking, whenever they feel free to choose or decide, such inner experience of freedom is infallible. No introspection or self-knowledge is required to experience or identify such freedom, however determined or motivated, just as no introspection or self-knowledge is needed to be in pain, namely, to experience pain, regardless of the grounds that determine it. Suppose that some chemical factors are the grounds for our feeling free or experiencing free will. Whether we know of such grounds and of their impact on us or not, the feeling or experience of such freedom is infallible and should not be considered as illusion at all. Equally, feeling well, comfortable, relaxed, and the like by virtue of such chemical factors

should not be considered illusion at all. The feeling or experience is certainly real and it is not about another reality except for that of the feeling or experience itself.

Hence, contrary to Spinoza and others, we should not ascribe the alleged “illusion” of free will to our ignorance of the causes that actually have necessarily determined our volitions and thus allegedly made them not free at all. Spinoza’s view on the illusion—“fiction” and “error” in his explicit terms—of free will deserves special attention. If we consider carefully the main arguments in the *Ethics* according to which free will is merely a fiction or an error based upon the ignorance of the causes that determine our volitions,<sup>9</sup> we can realize that the reality of free will should not be deemed an error provided that we do not also follow Spinoza’s actualism and psychophysical stance. Illusion or error consists of considering a fragment of reality as if it were a reality *in se est*, a complete piece of reality, which is not the case at all. For example, if we perceive a stick as broken once it is put into the water, we make no error in perceiving it as broken. But if we jump into the conclusion that the stick *in se est*, namely, as it is in itself, is broken, we certainly err. The sun appears to us as small as our hand, and no error occurs when we see it as such as long as we do not believe the sun *in se est* to be as small as our hand. Thus, such subjective experiences are emendable fragments of reality under Spinoza’s meticulous examination. The illusion, fiction, or error enters the scene whenever we ignore the limited, dependent, and conditioned nature of that experience as such a fragment.

Under Spinoza’s examination no isolated fragment of reality exists, for any detail or fragment of reality inseparably pertains to the reality as a whole. Each such detail or fragment is simply a link in a total causal chain or unbroken series, which is nature as a whole under this or that Attribute. Each causal link is thus necessarily, inseparably connected to all the others. While under ignorance, illusion, or error, we are not aware of such a necessary inseparable connection and we refer to the fragment of reality as if it were a discrete, unconditioned, or isolated part of it, as an island existing for its own, which Spinoza regards as sheer absurd. According to him, nothing except total reality is entitled to be considered unconditioned.

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<sup>9</sup> See Note 8 above.



Whenever we realize that our subjective experience, such as that of free will, is not an isolated, independent, or unconditioned piece of reality, we cannot err about it and we know for certainty that no volition can be uncaused. Each volition is simply a link in a necessary causal chain, all of whose links are subject to strict determinism. Once we realize the causes of our volitions, we cannot err about their nature as necessarily determined. Spinoza assumes that this makes our volition not free. In any event, such error is emendable, as in the complete context of the total reality no room exists for error or illusion. As long as we do not consider the part as if it were a whole, no error or illusion can take place. Indeed, each of our mistakes, errors, and illusions is necessarily caused, and each necessarily takes part in nature, in the reality of things. It ceases to be an error or illusion once we realize its partiality and the causal connections that link it to the whole of nature.

Now, unlike illusion or error, ignorance of the causes in the case of free will or pain does not lead to any illusion or error as to the reality of pain or of free will. Whether I know what are the causes of my pain or not, its reality is undeniable. When I am entirely ignorant of the causes of my pain, I am still undeniably in pain. From the total view, in which no error has any room, *sub specie aeternitatis*—from the point of view of the infinite intellect—any *such* experience, despite its undeniable subjectivity, is a necessarily real piece of reality. Which means that even from a point of view that conceives all the relevant reasons for and causes of such experiences, such experiences remain true with no change as to their epistemic status. Such mental states enjoy the status of adequacy, in which the same truth is equally valid for the parts and the whole, which is the case of any adequate or rational knowledge in Spinoza's view. According to such view yet contrary to Spinoza's explicit conclusion, free will<sup>10</sup> is real as well as being subject to adequate knowledge. As *adequate* parts of reality, mental states such as being in pain and of free will are not mistaken at all and they should not be considered errors, fictions, or illusions. Spinoza could not argue that pain is an error, fiction, or illusion, even to the extent that we do

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<sup>10</sup> More precisely, free volition, for, according to Spinoza, will, especially free will, is merely an illegitimate abstraction or universal pertaining to the first kind of knowledge, *imaginatio*. Yet, since I use “will” as the common *property* of all volitions, I use it as an adequate term in the second kind of knowledge—*ratio*—legitimately referring to the common properties of entities.

not know the causes of it. I venture to argue that the same holds for the experience of free will. As much as the experience of pain is being in pain, which Spinoza would not deny, the experience of free will is an adequate mental reality, which he *should* not deny or deem as an illusion. Pain is not an illusion believed to be an unconditioned, discrete, or isolated part of reality; it is necessarily connected to the whole of it. We are in pain inescapably under some circumstances as much as we feel ourselves as having free will under some circumstances. The experience of pain enjoys a secure adequacy in the reality as a whole, and from no point of view or perspective can it be doubted as if it were an error, fiction, or illusion. The same should hold for our experience of free will as a mental reality. Thus, Spinoza's analogies to the alleged "illusion" of the stone as to its "free" fall and to alleged "free" desire of the baby to be breast-fed are not valid for the experience as well as the reality of free will.

Furthermore, as much as being in pain is not subject to the Spinozistic emendation, which requires a knowledge of the causes of a fragment of reality, the experience of free will is not subject to any emendation that could turn it from illusion or error into true knowledge. After all, just like being in pain, the experience of free will is infallible. Hence, becoming aware of the comprehensive, complete causal context of any such experience does not affect the infallibility of any of them. The explication of the relevant causes of both experiences does not change the nature of the experience itself, which need no emendation, for from the outset it has been fallible and could not be an error, fiction, or illusion. The causal context does not change the epistemic status—the veridicality or the adequacy—of such an experience even slightly. Unlike the optical illusions of the broken stick, the smaller or the nearer sun, no fragmentation or imaginative isolation is involved in the nature of the experience of pain or of free will. Most significantly, being in pain and the experience of free will should not pertain to the first kind of knowledge—*imaginatio*—which Spinoza deems as the origin of any error or illusion. They pertain instead to the adequate kinds of knowledge, namely, *ratio* and *scientia intuitiva*.

Once we conceive the possibility that freedom of will and determinism, causal or otherwise, are compatible, nothing about free will remains under illusion or error. Given that nothing in reality is without cause, and even the reality as a whole, as a totality, is caused (in this case, it is the cause of itself, *causa sui*), each volition or the will in general is causally

determined. Nevertheless, contrary to Spinoza, this in itself does not make the will not free. One of the reasons that Spinoza could not reach such a bold conclusion lies in the fact that he was a confirmed actualist. Hence, pure possibilities do not exist in his ontology. If alternatives to any of our decisions are pure possibilities, no such alternative can exist in Spinoza's world, which entails that none of our decisions can be free in his view.

The question of the relevant causes, grounds, and reasons to the determination or motivation of the will is entirely irrelevant to the question of whether its freedom is real or merely illusion. The experience or feeling of inner freedom, of the freedom of our will, is absolutely not subject to any illusion, self-deception, or fallibility, just as being in pain is not subject to any of these, no matter what are the reasons or grounds for such experiences. Suppose that, like phantom pains, "phantom experiences" of free will exist, which means that such experiences have no grounds in external, intersubjective or objective, reality. Nevertheless, such experiences are as real as any experience of free will that has grounds enough in external reality and that is not considered "phantom" at all. Like pain, the experience or feeling of free will *is* an inner, mental reality and it is not about external reality.

"Inner reality" involves no "ghost in the machine" or anything of a similar fallacy. By "inner" I mean something mental or subjective, which is irreducible to any other kind of reality. Once you acknowledge mental reality, you have to acknowledge subjectivity too. Thomas Nagel has contributed greatly to our understanding of that (especially in 1986). But, again, I do not see why the case of pain and pleasure (*ibid.*, pp. 156-162), in which "no objective view we can attain could possibly overrule our subjective authority in such cases" (*ibid.*, p. 158), should not equally hold for our experience of free will. Be that as it may, without subjects and subjectivity, no mental reality exists at all. I use "reality" in the irreducible sense of the term, which means that mental-inner-subjective-personal reality should be accepted as real from any possible perspective: personal-subjective, interpersonal-intersubjective, or impersonal-objective. Intersubjective reality is the social, communal, national, political, linguistic, or communicative life (or "form of life") that one shares with others. Objective reality, including one's body, is the physical reality in which one exists. As a person, one is a mental being, actualized as a body, which takes part in the physical, objective reality.

The later psychophysical assumption should not be considered dualistic; it simply commits itself to a psychophysical irreducibility. Note especially that any psychophysical distinction, which is entailed by the psychophysical irreducibility, does not lead to psychophysical separation. As much as the mind is irreducible to the body and vice versa, subjective, mental reality is irreducible to objective or intersubjective reality. The reality of pains, volitions, emotions, feelings, and other mental states is, inescapably and irreducibly, subjective; yet it bears intersubjective or objective significance, which is as real as the subjective.

Feeling myself mentally free is as real and infallible as feeling myself well, unwell, in pain, relaxed, calm, peaceful, comfortable, uncomfortable, excited, tense, strained, and the like. Any adult is capable of infallibly distinguishing between such states of mind. Who on earth can repudiate my answers to the questions—“How do you feel?”, “Do you feel free to decide...?”, “Did you do it out of your free will?”, and the like—whenever there is no suspicion that I do not inform about my feeling bona fide? The experience of inner freedom must be infallible, whatever are the grounds, causes, or reasons for it, and nothing can disavow it as real. Unlike illusion, delusion, or hallucination, such experience is both real and infallible intersubjectively or objectively. As far as experience such as having free will or being in pain is concerned, the only authority is the person who has it. No intersubjective or objective authority can overrule it.

Is not James, whose cerebral damage has permanently paralyzed his left hand, under an illusion or self-deception whenever he feels free to raise it? He is certainly under an illusion as to his physical capability. Yet, despite his physical state that does not allow him to raise his left hand, James’s free volition or decision to raise it (or his attempt to do so) is by no means an illusion. Such a wish or decision is a “phantom” experience taking part in his mental, inner reality and, as such, it is absolutely real, not for James alone, but also for anybody else, since James’s mental reality as subjective should be intersubjectively and objectively acknowledged (as in the case of phantom pain). Nobody can rationally or intelligibly challenge the reality or infallibility of such an experience. James certainly disavows his *physical* state, which is quite common in some cases of cerebral damage, but surely he does not deny or disavow his conscious volition or decision whose reality is of mental freedom. James’s awareness of this inner free-

dom is infallible. Were he coerced to want or to decide to raise his left hand, he would have been aware or conscious of such coercion. He has no illusion about his will as mentally real and free.

I deem that all those who have considered freedom of will merely illusion or non-reality, have, in fact, referred to belief or knowledge about our will. But this must not be the case. First and foremost, freedom of will is an experience in which the perceived reality and the perception of it are one and the same. The experience of free will is not any kind of knowledge. The question, "How do you know that your will or choice is free?" is as absurd as the question, "How do you know that you are in pain?" The experience of either pain or free will does not depend on any knowledge. It is subjective, personal, and private. Knowledge, by contrast, is an intersubjective or objective matter. Hence, since my experience or feeling of free will reflects on the intersubjective and objective reality that we share, the intersubjective or objective bearing or significance of my experience is subject to knowledge. Since no knowledge is infallible or beyond any possible doubt, one can be mistaken about the meaning or significance of one's feelings or experience, as far as intersubjective and objective reality is concerned. But such fallibility, such capability of mistaking, does not hold for the subjective reality, yet reality by all means, of one's experience or feeling of free will. As with phantom pain, persons may mistake and be wrong as to the objective or intersubjective *significance* of their truthful experience or feeling. The objective significance is about one's physical state and behavior; the intersubjective significance is about one's attitude, language, expressions, and relationships. In intersubjective or objective reality some persons may not appear free at all despite their feeling or experience. But, just like their sense of pain, their sense of inner freedom, unlike their sense of objective or intersubjective reality, is free from any illusion or self-deception.

In conclusion, from any of the aforementioned possible perspectives (subjective, intersubjective, or objective), each person is the only authority as to her or his sense of free will. The significance of such authority is certainly intersubjective and objective. Whenever persons experience or feel themselves as having free will, no one, however capable or knowledgeable, can disavow such feeling or experience and consider it merely an illusion or self-deception.

That our experience of free will is not an illusion does not deem our will unmotivated or undetermined. How to render determinism, responsibility, and the necessary connection between a person and her or his volitions or decisions coherent or compatible with free will is one of the most intricate philosophical problems, if solvable at all (Kane, 2002, pp. 3-41). Peter van Inwagen, for one, highlights the unsolvable or dissolvable mystery of free will (1993, pp. 184-199, and in Kane, 2002, pp. 158-177). He concludes that free will remains a mystery: though it “undeniably exists, ... there is a strong and unanswered *prima facie* case for its impossibility” (ibid., p. 159), given that free will is considered incompatible with determinism and indeterminism alike. But the philosophers’ incapability of adequately solving such intricate problems, if solvable at all, does not repudiate or disavow the reality of free will. We are still lacking greatly in understanding the phenomenon of pain, but this should not make any of us question the reality of pain. Analogously, the assumed failure or inadequacy of any known explanation to the reality of free will does not repudiate this reality at all. Undoubtedly, there are reasons or grounds for the motivation and determination of any responsible person’s will, yet it is undeniable that the will is *both* free and motivated, even necessarily or inescapably motivated. If no philosophy can explain this, at all or adequately enough, the reality of free will is, nevertheless, undoubtedly there, simply in the heart of the mental life of each of us. Elsewhere, I have suggested a novel possibilist solution to that problem (Gilead, 2003, pp. 131-156), but even if no solution existed, the reality of free will should not be questioned, let alone repudiated or disavowed.

My view on the reality of free will opposes any “free will subjectivism,” such as Richard Double’s (in 1991 and in Kane, 2002, pp. 506-528). Given that mental, subjective *reality* is irreducible, and given that it has room enough side by side to intersubjective or objective reality, free will is undeniably real. This means that, metaphysically or ontologically speaking, *in fact* persons *really* have free will, and the reality of their free will should be acknowledged from any possible perspective, despite the difficulties or unsolved problems it may raise for philosophical or scientific thinking. In other words, to consider free will as an illusion, mistake, or self-deception is itself an illusion, self-deception, or mistake, for the fully-fledged reality of free will is an undeniable fact about persons or mental beings, equal to pain and other mental, subjective states. As much as the reality of pain is essential to our survival, the reality of free will, not an illusion of free will,

is essential to our human reality and life as well as morality. The reality of free will is independent of the perspectives in which it may be captured. We should be ontologically committed altogether to this reality. Furthermore, in the case of free will or pain, objective or intersubjective reality supervenes on subjective reality, for the latter is the ultimate authority as far as the reality of free will and pains is concerned. Being real from the subjective or personal perspective, they should be treated as real from the other perspectives, for no illusion of pain or free will is possible. They exist side by side to intersubjective and objective reality, independently of the standing of our knowledge or beliefs.

In sum, any denial of the reality of free will is as irrational or groundless as any denial of the reality of pain. The experience of free will is by no means an illusion. Such an illusion is merely impossible.

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SORIN BANGU

**Later Wittgenstein  
on Essentialism, Family Resemblance and Philosophical Method<sup>1</sup>**

ABSTRACT

In this paper I have two objectives. First, I attempt to call attention to the incoherence of the widely accepted anti-essentialist interpretation of Wittgenstein's family resemblance point. Second, I claim that the family resemblance idea is not meant to reject essentialism, but to render this doctrine irrelevant, by dissipating its philosophical force. I argue that the role of the family resemblance point in later Wittgenstein's views can be better understood in light of the provocative aim of his philosophical method, as stated (for instance) in PI 133: "[t]he philosophical problems" - associated with essentialism in this case, "should *completely* disappear".

*Introduction*

In the paragraphs 65 to 67 of his *Philosophical Investigations*<sup>2</sup> (1953), Wittgenstein introduces his celebrated family resemblance point<sup>3</sup>. The example of games illustrates the claim that certain phenomena do not have "one thing in common which makes us use the same word for all - but they are *related* to one another in many different ways" – these similarities are characterized as 'family resemblances'. Thus, insisting that "There must be something common, or they would not be called 'games'" is nothing but

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<sup>1</sup> I thank to John Canfield and Oswald Hanfling for their helpful comments on earlier drafts of this paper.

<sup>2</sup> Hereafter PI, and the number of section. The citations from Wittgenstein's writings follow the usual notations: PG for Wittgenstein (1974), BB for Wittgenstein (1964), AWL for Wittgenstein (1979), Z for Wittgenstein (1967).

<sup>3</sup> The family resemblance point is ubiquitous in Wittgenstein's writings. Philosophically important concepts (such as 'proposition', 'language' and 'number', 'understanding' or 'believing') are family resemblance concepts. See, for example BB: 17-20, 33, PI 65 – 8, 108, 135, 179, 236 etc., Z 26, PG: 112, AWL: 96, etc.

prejudice, in so far as, if we ‘look and see’, we do not find any feature common to all games in virtue of which we use the same word for all.

Although Wittgenstein’s (anti)essentialism and the family resemblance point were consistently debated in the past, they receive almost no attention in the recent literature. One possible reason for this lack of interest is that these topics seem quite transparent now. A sort of silent consensus dominates the scene: fundamentally, Wittgenstein gets engaged in the traditional metaphysical dispute on essentialism (or ‘universals’), and claims, against essentialism, that there is no essence, no common property, no definition of games. The anti-essentialist interpretation is widely spread among scholars and constitutes, in fact, the standard reading of the family resemblance point<sup>4</sup>. However, in addition to the overt anti-essentialist position, H. –J. Glock’s *Wittgenstein Dictionary* (1996: 120-2) records a different interpretation of these passages. According to this second reading, more caution in attributing such straightforward anti-essentialist tenets to Wittgenstein is recommended. Baker’s and Hacker’s *Analytical Commentary* (1992:131) and O. Hanfling’s *Wittgenstein’s Later Philosophy* (1989:67), for example<sup>5</sup>, cast doubts on the first reading, arguing that Wittgenstein’s point is not that words lack essentialist definitions. Rather, the point of the family resemblance passages is that words *need not* have essentialist definitions (capturing common properties) in order to function as words.

In what follows I reexamine these two readings and I sketch a new approach to the family resemblance sections. More specifically, I maintain that the first reading (straightforwardly anti-essentialist) is at best simplistic and should be rejected; although I agree in spirit with the second reading, I shall argue for a revision of it. This revision consists in proposing a more precise formulation of Wittgenstein’s point. The primary virtue of this new formulation is that it explicitly rules out a certain interpretation of the second reading, interpretation according to which this reading is highly misleading. My reading is neutral with respect to essentialism and, in a sense to be explained, is weaker than the first two interpretations; yet, Wittgenstein’s point, as I’ll reconstruct it here, remains considerably strong. On my account, the family resemblance idea is not meant to reject essentialism, but to render this doctrine irrelevant, by dissipating its phi-

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<sup>4</sup> See, for example Pitcher (1964), Bambrough (1966), Malcolm (1986: 236-7), Ackerman (1988: 82-3), Rundle (1990: 41), Jaquette, (1998: 241-52), Lugg (2000: 115). The list could be much longer.

<sup>5</sup> Kenny (1973: 163) reads the passages this way as well.

losophical force. The role of these sections can be better understood in light of the provocative aim of Wittgenstein's philosophical method, as stated (for instance) at PI 133: "[t]he philosophical problems" - associated with essentialism in this case, "should *completely* disappear".

On my account then, the apparently well-understood family resemblance point should be read in a metaphilosophical key. Part of my project in this paper is to challenge the almost unanimous opinion according to which Wittgenstein's metaphilosophical views are rather incoherent, or even mistaken altogether. Highly idiosyncratic, his views on the therapeutic nature of philosophy are considered "the weakest part of Wittgenstein's later work – slogans unsupported by argument (...), which can be isolated from the rest"<sup>6</sup>. I maintain that these characterizations should be resisted; to the extent that my account of Wittgenstein's position on essentialism is convincing, it can render the connection between what he is doing and what he is saying about what he is doing (his aims and methods) more perspicuous.

### *The first interpretation*

Traditionally, essentialism claims that things have two different kinds of properties: some of them are essential, and the object must possess them to be what it is, while others are just accidental. Unlike recent essentialist doctrines (which employ the tools of various systems of modal logic to distinguish between different kinds of necessity statements<sup>7</sup>), traditional essentialism illustrates the "definitional conception of essence"<sup>8</sup>. According to this type of essentialism, the essential properties (which, when put together, presumably constitute *the* essence) of a term T are captured by the *analytical definition* of 'T'. The definition mentions those properties that are both necessary and sufficient for T to be what it is. A good example (not surprisingly found in a formal language) can be the analytical definition of 'even number': for every n, n is an even number if and only if (n is a natural number and n can be divided by 2). As it is evident, however, Wittgenstein's discussions in PI 65-71 are related to definitional essential-

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<sup>6</sup> As H. – J. Glock records in (1996: 294)

<sup>7</sup> One well-known distinction I have in mind here is, of course, between necessity *de dicto* vs. necessity *de re*. Another is between necessary properties as applied to individuals vs. applied to kinds. None of them plays any role in this paper.

<sup>8</sup> For the distinction between the definitional and the modal conceptions of essence see Yablo (1998).

ism, since he repeatedly addresses issues concerning the definition of a concept. For this reason, I shall discuss here only this version of essentialism<sup>9</sup>. To begin with, let me outline three interpretations of Wittgenstein's view on definitional essentialism I focus on in this paper.

Assuming that the target of PI 65-67 is definitional essentialism (there is a common feature, a definition of games), the first interpretation summarizes the anti-essentialist reading, straightforwardly denying definitional essentialism:

(1) There is no analytic (essentialist) definition that captures the common feature ('essence') of games.

Hanfling (1989: 67, 2002: 90) and Baker and Hacker (1992: 131) read the family resemblance point in PI differently:

(2) A concept-word like 'game' does not need an essentialist definition of games in order for speakers to apply 'game' correctly.

The third reading will be argued for in this paper:

(3) Speakers do not need to know an essentialist definition of games in order to apply 'game' correctly.

A few preliminary remarks on these three readings are in order. The first, straightforward anti-essentialist interpretation takes (1) to be Wittgenstein's point in the family resemblances passages. I'm going to reject this view. Thesis (2) outlines the second reading and, although I'm rather sympathetic to it, I'll argue that it can be misleading. I propose (3) as expressing Wittgenstein's point in the family resemblance passages.

Proposition (1) is what is usually called an ontological thesis. It is a thesis about the (non)existence of an essential, common property. Proposition (3) is, of course, not an ontological thesis; it is rather an *epistemological* point. It tells us about what speakers need to know in order to use a word. As I'll argue, (3) is meant as a *description* with a significant philosophical (therapeutic) relevance. In order to confirm its accuracy, we have to look at the use of words and examine what speakers do when they apply them. Nevertheless, it may be misleading to speak here about confirmation

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<sup>9</sup> Note that I do not address in this paper Wittgenstein's very important claim in PI 371, 'Essence is expressed by grammar'.

or refutation, since, as we shall see, thesis (3) is not meant as a (scientific) hypothesis. Let me address the first interpretation now. The next section will be devoted to the second reading.

As noted, commentators do not usually distinguish between theses (1), (2) and (3) and claim that Wittgenstein advanced the first thesis. A recent sample of this reading is D. Jacquette:

Wittgenstein illustrates the failure of essentialist definitions to identify the essence of the concept game (...). The class of things we call 'games' is so diverse and open-ended that we cannot arrive at any common set of distinguishing properties. (1998: 241)

After quoting PI 66, Jacquette goes on and points out what he believes is Wittgenstein's underlying point:

The empiricism of Wittgenstein's later philosophy is evident. Wittgenstein enjoins us to look at the world in order to decide whether or not an essentialist definition of the concept game is possible. (1998: 242)

Although Jacquette's emphasis on a certain empirical aspect in Wittgenstein's strategy is not completely beside the point (I shall clarify later why this point about Wittgenstein's empiricism is still ambiguous), an obvious objection plagues this reading. If Wittgenstein's thesis were (1) - the ontological one, then Wittgenstein's empirical procedure ('to look and see') was not appropriate for supporting a thesis like that. If one looks for the essence of games, for a definition of games, and one does not find them, then this failure proves nothing. If one does not find what one looks for, then there are always two explanations of the failure: either there is nothing to be found indeed *or* one does not look at the right places in the right way. Why should essence be that kind of thing that could be found by following the method proposed by Wittgenstein – by 'looking and seeing'?

It is hard to believe, then, that the method Wittgenstein seems to advance here, namely to *look* at how things *are* and then take note, *describe* what we actually *see*, can produce any persuasive outcome. I emphasize this point because this objection is directly relevant to one of his main metaphilosophical thesis, according to which "philosophy really *is* purely descriptive" (BB: 18), hence "we must do away with all *explanation*, and description alone must take its place" (PI 109). As the above objection shows, by proceeding in light of these claims, Wittgenstein cannot yield any philosophically convincing result, hence the almost unanimous dis-

missing attitude toward these views. However, as I'll argue, these views on the philosophical virtues of descriptions deserve, in fact, much more credit when put in relation to a correct interpretation of these passages.

I think then (as Baker and Hacker claim too<sup>10</sup>) that the above objection is very convincing; moreover, no textual evidence exists to the effect that Wittgenstein answers it, although he is aware of it. In (BB: 18, 35), for instance, he warns that, due to her following the method of the natural sciences, the essentialist tends to think that a question such as “What is the definition of ‘game’?” has the same status as a scientific question. When the essentialist is faced with the failure of finding a common feature of games (to be captured in a definition), she replies that, as it often happens with some scientific questions, no answer has been discovered *yet*. And, if no definition has been formulated yet, it simply doesn't follow that a definition does not exist. This simple reasoning should be enough to show that, if Wittgenstein were to hold (1), then this would be a very weakly supported claim, worth of little philosophical interest.

### *The second interpretation*

I argued that one misleading way to read Wittgenstein's family resemblance point in PI was to claim that he endorses thesis (1), thus failing to distinguish between theses (1), (2) and (3). As I noted earlier, Baker and Hacker (1992) and Hanfling (1989, 2002) dismiss thesis (1) as capturing Wittgenstein's point. According to Baker and Hacker (1992: 131) the correct reading is as follows:

[Wittgenstein refutes] the philosophical dogma that a concept-word is correctly applied to each of a set of objects *only if* these share some common feature in virtue of which they fall under this concept. (Italics added)

Hanfling's reading highlights the contrast with the first anti-essentialist reading (1989: 67):

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<sup>10</sup> Baker and Hacker (1992: 131) write: “Does Wittgenstein prove there is *nothing* common to all games? That we can *never* discover a common property? By running through various kinds of games, he marshals inductive support for this negative existential statement but might it not be refuted by a more penetrating analysis of games? His claim seems precarious, but also unnecessarily strong.”



The crucial issue is not whether words have (...) an essentialist definition, but whether they *must* have one, in order to function as words. It is the second claim that Wittgenstein denies<sup>11</sup>.

The two passages above outline a different reading of Wittgenstein's point, summarized in Hanfling's (2002: 90) statement:

[T]here *need* not be... such set of conditions [an analytic definition] (...). [A] word can function perfectly well without this support.

On my understanding of their views, Hanfling, Baker and Hacker seem to think that Wittgenstein's point is thesis (2), outlined above, which amounts to this:

(2) It is not necessary that a concept-word have an essentialist definition in order for speakers to apply that concept-word correctly.

Or equivalently: we can find (some) words that lack essentialist definitions (since things lack common, essential features to be captured by these definitions); however, despite that, speakers use them correctly.

Several remarks are in order. Note, first, that this reading is a serious improvement to the simplistic thesis (1). Thesis (2) emphasizes not only the lack of an essential feature (as thesis (1) does), but also *the role* this feature is meant to play in the use of the word. The Baker-Hacker-Hanfling interpretation correctly underscores the crucial aspect here, namely that Wittgenstein does not merely and dogmatically deny the existence of a common feature. In his view, this denial should not be separated from the role this feature is meant to play in speakers' use – to “make us use the same word for all” (PI 65). This second reading illustrates the fact that the role assigned to the *use* of the word is fundamental for the later Wittgenstein's philosophical methodology.

My main concern about this reading is that, despite the new and correct emphasis on use, it may not completely succeed in avoiding the misleading suggestions made by thesis (1). This is apparent when we interpret thesis (2) as follows. A defender of thesis (2) has to present some cases in which both clauses present in thesis (2) hold, namely:

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<sup>11</sup> Similarly, Glock (1996: 121) remarks: “[The] qualms about the claim that games have no common defining characteristics [see footnote 11] leave intact the more modest claim that they *need not* have any such thing in common [on account of which speakers apply the word ‘game’].”

- (i) to present some words that lack essentialist definitions (or, equivalently, a class of things that lack a common, essential feature)

and

- (ii) to show that speakers can use those words even in these circumstances.

Games serve here as an example satisfying both these clauses.

Let me note two things about clause (i). First, clause (i) is equivalent with thesis (1) and, for this reason, thesis (2) presupposes thesis (1). Second, clause (ii) is subaltern to clause (i): thesis (2) reads, in fact, “it is possible that (some) words lack essentialist definitions and, *even in these circumstances*, speakers can apply them correctly”. However, clause (i) is, as we saw, very problematic. No proof can be convincing for the essentialist: even if nobody has found a definition of ‘game’ yet, this does not prove that a definition does not exist or won’t be found in the future. If no way to defend a clause like (i) is available, then, in so far as thesis (2) presupposes it, no strategy to defend thesis (2) can be very promising as well.

Therefore, despite its merits, thesis (2) seems to pose the same difficulties as thesis (1); hence its defense is no less problematic. These difficulties stem from the fact that one’s commitment to thesis (2) appears to entail one’s commitment to clause (i). To be sure, I’m not claiming that Hanfling, Baker and Hacker have intended this entailment<sup>12</sup>. Yet I’m claiming that thesis (2) is ambiguous; as it stands, an interpretation of thesis (2) is possible that suggests this problematic entailment. Consequently, precautions should be taken to the effect that the above interpretation (involving clause (i)) is ruled out.

As I suggested when I motivated my proposal of thesis (3), my view is that Wittgenstein *does not even address* thesis (1); he simply does not engage in a dispute over it. Moreover, as we’ll see, the interesting philosophical point he makes in those passages does not depend on his direct refutation of definitional essentialism. My reading is slightly different, being captured by the following thesis (put in a form similar to thesis (2) but equivalent to thesis (3) above):

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<sup>12</sup> Hanfling confessed (in personal correspondence) that his intentions were along the lines of thesis (3). He denied any relevant difference between thesis (2) and (3).

(3) It is not necessary that *speakers know* essentialist definitions in order to apply words correctly.

Or, equivalently: even if the speakers don't know the essentialist definition of a word, they are nevertheless able to use it correctly. I assent, therefore, to a revised version of the second reading. In Hanfling's case, my reading runs like this: a word can function perfectly well without speakers knowing how to formulate its definition<sup>13</sup>. In the Baker-Hacker version, the modification I propose is similar. Their point should be understood as follows. The philosophical dogma that Wittgenstein refutes is that a concept-word can be correctly applied to each of a set of objects only if speakers know the common feature shared by these objects, in virtue of which they fall under this concept. I thus maintain that Wittgenstein's concern is not related to what must be (objectively?) true about *concepts* - namely, that their use is conditioned by their having an essentialist definition, but to what *speakers* actually know in order to use them. My proposal restrains the second reading to what is accessible to speakers; it also explicitly rejects the suggestion that Wittgenstein held a sort of substantial (negative) thesis about how the relation between language and world ('there is no definition of games') is reflected in speakers' linguistic behavior ('despite that, speakers can use the concept').

Thesis (3) is, however, weaker than thesis(1), since (1) entails (3) and (3) does not entail (1). If, in some cases, ('objectively') there is no definition of a concept (i.e. 1), then, obviously, speakers cannot know it, hence it cannot be the case that to know the definition is necessary for the correct use of the concept (i.e. 3). But this entailment raises no difficulties, since thesis (3) is not defended on the basis of (1). Thesis (3), as we saw, is not inferred from a prior proposition, rather it is endorsed by descriptions of the speakers' linguistic behavior. On the other hand, (3) contains no reference to what is, so to speak, 'objectively' the case as regards the existence of definitions. Essentialist definitions may or may not exist, thesis (3) remains silent on that; it just states that knowledge of definitions is not mandatory for a correct usage. That is, (3) neither entails nor contradicts (1), i.e. the truth of (3) does not rely on (1). (Although (1) entails (3), if (1) is false, (3) can still be true.) Although (2) makes a statement with respect

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<sup>13</sup> Naturally, this is not to say that speakers can use a natural language word without being able to offer any justifications (such as clues, resemblances etc.) as to why this use is appropriate. What is denied is the knowledge of a specific definition such as that available for formal concepts like 'prime number' or 'denumerable set'.

to the role the (existence of the) definitions play for speakers, the defense of thesis (3), unlike that of thesis (2), does not involve thesis (1). (We saw that according to the interpretation I sketched above thesis (1), being equivalent to clause (i), is in fact part of thesis (2).)

Summing up, the main gain in accepting thesis (3) as the correct interpretation of the family resemblance point is that the 'defense' of thesis (3), unlike that of thesis (2), makes no appeal to the validity of thesis (1). Thesis (3) possesses then a virtue that (a certain interpretation of) thesis (2) is lacking, namely the independence from the highly problematic thesis (1). Whether or not thesis (1) is true or false, what thesis (3) says is still valid. Thus, thesis (3) is *neutral* with respect to thesis (1). Note, however, that although (3) does not reject essentialism, the effect of (3) on it is no less philosophically relevant: (3) says the existence of a common feature has *no function* in our use of the word.

As I see it, Wittgenstein's main point is – no surprise - about speakers' *use* of words. He wants us cured of the assumption that there must be a requirement imposed on us, on speakers, requirement consisting in being able to point out to the essence of games while we use the term 'game' correctly. In the reading I advance here, the role of use is emphasized in the second clause of thesis (3): first, speakers don't know the definition; second, they use the word correctly. As Wittgenstein urges frequently, by paying attention to speakers' everyday use of natural language concepts we can see that we do not feel, in fact, the pressure of the requirement to be able to identify a common feature while we use the terms correctly. It first looks like we do need to meet this requirement. Yet, when we *really* look at our everyday use, we discover that we ought not feel, in fact, this need. (As Wittgenstein says somewhere, it is not "our real need"<sup>14</sup>). The essentialist argues that the requirement 'objectively' exists, and she, *qua* metaphysician, feels its constraint, its (metaphysical) pressure. Despite that, natural language speakers (including the essentialist *qua* speaker of everyday language) can confess that they do not feel the pressure of the requirement in the everyday use of words, since speakers do not need to identify a common feature in order to use the term 'game' (for instance) correctly. Thus, by looking at use in a certain way, we ought to discover – Wittgenstein urges - that this pressure has a curious status: it is like a need that we,

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<sup>14</sup> This is the sense in which I take Stanley Cavell's (1979: 187) point: "But I think that all that the idea of "family resemblances" is meant to do (...) is to make us dissatisfied with the idea of universals as explanations of (...) how a word can refer to this and that and that other thing, to suggest that it fails to meet 'our real need'."

as speakers of natural languages, do not feel. Therefore, we should ignore this supposed requirement, we can dispense with concerning about it. When this happens, to use Wittgenstein's own terms, the 'therapy' succeeds, we do not feel that 'metaphysical' pressure anymore.

*More on the proposed interpretation*

Following Hanfling, and Baker and Hacker, I concluded that it is very implausible that thesis (1) expresses Wittgenstein's point. Nevertheless, the second reading inherits the difficulties of the first interpretation (thesis 1) since thesis (2) assumes thesis (1) in the form of clause (i). I proposed thesis (3) as a revision of this second interpretation, thus trying to explicitly emphasize the crucial role *speakers' (everyday) use* of the words has in Wittgenstein's view. Now I make a few more remarks on the nature of thesis (3) and on how it squares with later Wittgenstein's overall (meta)philosophical views.

Characteristically, Wittgenstein's main strategy to defend thesis (3) consists in asking us to pay attention to 'what is going on' when we use a word. It is this concern, I contend, that gets addressed in the family resemblance passages too. Like in many other places throughout PI, Wittgenstein's main point in directing our attention toward this aspect is to make us realize that there is nothing that constantly and mysteriously *accompanies* our use of a concept. As a matter of fact, we do not (unconsciously) identify a common feature of games and we do not have an essentialist definition in mind while we use the word 'game' correctly. Therefore, as a matter of *fact*, we need not identify some common feature when we use a word - that is exactly thesis (3).

It is worth noting that Wittgenstein's way to proceed in PI 66 is, in fact, an illustration of his overall philosophical strategy, summarized in PI 127: to assemble reminders for a particular purpose. (Note that this view belongs to the aforementioned group of idiosyncratic statements regarding the nature of philosophy as well). Specifically, he urges us to remind how we use the word 'game'. Did we identify the common feature in virtue of which we applied the word to card-games? Or to board-games? Did we appeal to any exact definition that would capture that common feature? His answer is definitely 'no'. Our approval of the description performed by thesis (3) is meant to be immediate: we really do not know any suitable definition of 'game', we simply cannot identify that feature; notwithstanding this, we can use the word appropriately. This makes his therapeutic

purpose clear: to render *visible* that we do not need to assume the (epistemic) burden of knowing the common feature when we apply the word correctly.

Let me add two points to clarify what kind of statement is proposition (3) and a third point to explain its genealogy. The first issue (perhaps only superficially problematic) is the fact that I capture Wittgenstein's position by formulating a sort of (philosophical) *thesis*. Given Wittgenstein's well known rejection of explanations and philosophical theses made clear in PI 128<sup>15</sup> my term seems to be at odds with his explicit dismissal. However, as I highlighted it earlier, thesis (3) should be read as a *description*, as a way to take note of what is familiar and simple, being always before our eyes (PI 129); hence the word 'thesis' should not worry us here<sup>16</sup>. This is not a thesis in the sense that it states something worth defending, worth explaining by adducing further empirical evidence. Thesis (3) is not worth defending since nobody challenges it. We all know that we do not use any definition when we apply words like 'game' correctly, therefore there is nothing special with the remark that we do not need such a definition. We all know that what regulates our use of words is the way we learn how to use them in childhood, through comparisons, analogies, small clues etc., that is, a complicated mixture of explicit and implicit indications.

Secondly, let me make a few remarks on the labels I used to characterize thesis (3), namely that it is a *description* of what actually goes on in use, making an epistemological point. Being a description of the actual use, it may seem it is an *empirical* point. While I fully endorse the first label – thesis (3) is a description - I used the second one just for convenience: to say that (3) is an empirical statement is misleading. Let me clarify this, thus trying to clarify what is wrong with the above Jacquette's characterization of Wittgenstein's later philosophy as inspired by empiricism.

Wittgenstein does think that it is observations into "the workings of our language" that can support a thesis like (3). These workings are revealed in speakers' everyday linguistic practices. These practices are social practices, objects of empirical research for that matter. We do not infer the grammar of a concept from some prior principles, but learn language by getting involved in a number of paradigmatic situations of language use

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<sup>15</sup> PI 128: "If one tried to advance *theses* in philosophy, it would never be possible to question them, because everyone would agree to them."

<sup>16</sup> Nonetheless, what should worry us here is that our crispy manner of presenting Wittgenstein's view (by advancing and analyzing some theses) is not consonant with the colloquial, self-questioning spirit of the

(naming, describing, asking, supposing etc.) within these practices. This is the point at which the alleged Wittgenstein's 'empiricism' enters the scene: his grammatical investigations focus on the *actual* linguistic practices. So, on one hand we can say Wittgenstein displays an overall 'empiricist' inclination in directing our attention toward inspecting our use of concepts, toward the actual fact that we do not know and we do not employ any definition when we use the word. On the other hand, Wittgenstein's point is not (as Jacquette claims) that empirical inspections of games confirm us they share no common feature. "To look at the world", as Jacquette put it (in the earlier quote), is, in this context, hopelessly ambiguous. Because our use is part of the world in the sense that it is not a fiction<sup>17</sup>, one may be mistakenly lead to think that Wittgenstein's 'looking at the use' can be subsumed to 'looking at the world', and thus conforms to the traditional empiricists doctrines<sup>18</sup>.

This last point can be made even clearer if we recall the main feature of empirical statements, the possibility of being refuted by further empirical findings. What thesis (3) claims is not meant to be an empirical statement in the sense that it may be overthrown by further empirical investigations. Thesis (3) does not even belong to the domain of scientific, empirical investigation since it is not a hypothesis that has to be tested, it does not reveal a *new* fact, a new property etc. as scientific discoveries usually do. It is a (supposedly philosophically illuminating) description (PI 109), open to everyone's approval; it does not require for that any special instruments or laboratories. What thesis (3) says has always been, is and will always be before everyone's eyes, in a way in which scientific discoveries are not. That thesis (3) is endorsed by straightforward remarks about how we use natural language is in agreement with its 'philosophical' relevance in Wittgenstein's account: "[Philosophical problems] are, of course, not empirical problems", but they can be solved "by looking into the workings of our language" (PI 109). Now it is worth pointing out that this is exactly what the strategy to 'defense' thesis (3) amounts to: to look into these workings and to describe how speakers use the word 'game'. We can make, of course, empirical investigations (linguistic-statistical, say) regarding which

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*Investigations.*

<sup>17</sup> "We are talking about the spatial and temporal phenomenon of language, not about some non-spatial, not temporal phantasm" (PI 108)

<sup>18</sup> Hanfling (2000, ch.4) challenges the application of usual classifications (empiricism, rationalism, idealism etc.) to Wittgenstein's views. He proposes the term "participatory knowledge" for the kind of knowledge one acquires when one learns a language.

features of games are considered the most characteristic for games by some categories of speakers. There might be neurological patterns associated with the use of a certain word; it might turn out that only certain parts of the brain contribute to processing certain concepts, so far and so on. But to claim that further empirical investigations can reveal that we *do* know a definition of games in spite of our denial (namely, that we do not know and do not use any definition when we apply words like ‘game’) is to get entangled in a form of conceptual confusion. Thesis (3) is a then description with therapeutic power<sup>19</sup>, apt to disperse this confusion.

Wittgenstein’s discussion of ‘unconscious pains’ in (BB: 22-23) can be recalled here to explain in what sense we speak about confusion here. It is the conventions that govern the correct uses of the word ‘pain’ (its ‘grammar’) that rule out as meaningless to say we are in pain when we do not feel any pain. The emphasis on ‘grammar’ here is meant to underscore that this is not an empirical discovery. A scientific (medical) discovery can reveal, for example, that we have internal wounds which are not painful, but no scientific discovery can reveal we have pains which we do not feel. This is so not because pains have some mysterious causal relations to what we feel (beyond what science can bring out), but because of the grammatical relation between concepts like ‘pain’ and ‘knowledge’. We can speak, of course, about ‘pains we do not know we have’ and say we have these kinds of pain, for example, in the aforementioned case when some internal wounds are not painful. Yet, as Wittgenstein notes in BB, to speak this way is just to introduce new *terminology*, a new concept of pain and not to discover a new empirical fact about pains (i.e., that they can be such that we do not know about them.)

By the same token, no scientific discovery can reveal that, in spite of the fact that we realize we do not know any definition<sup>20</sup>, we *do* know a definition when we apply the word ‘game’<sup>21</sup>. Like “We are not in pain if

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<sup>19</sup> Recall one of Wittgenstein’s own conception of his enterprise: “Philosophy really *is* ‘purely descriptive’” (BB: 18).

<sup>20</sup> As we’ll see later on, Wittgenstein distinguishes between knowing of an essence of games and being able to capture it in words, by formulating a definition: speakers may know about an essence of games but it may turn out that it is ineffable: “it is only other people whom we cannot tell exactly what a game is” (PI 69).

<sup>21</sup> Of course, nothing precludes a community of speakers to propose a definition of ‘game’ and follow it strictly. For such a proposal see PI 76 and Rundle’s (1990: 48) (amusing) proposal of the following fifty three-word definition: “games are rule-governed activities with an arbitrary and non-serious objective, an objective that is of little or no significance outside the game, but which we set ourselves to attain for the



we do not feel any pain”, “We know what is going on when we apply words”<sup>22</sup> is not a point about how knowledgeable speakers are, a piece of factual information to be confirmed or refuted by empirical research, but a grammatical point. So being, it makes no sense to ask whether or not this is an empirical generalization, a sort of inductive reasoning. Moreover, the wonder how could Wittgenstein think such a statement endorses thesis (3) is out of question. (“Did he ask all speakers how they use words like ‘game’?”)

Thesis (3) has then a grammatical status; it is established on the basis of *descriptions* of the way we use the words and it is meant to direct our attention toward what everybody already agrees on. It does not state anything new for speakers, it is not a hypothesis, a prediction, but it is prompted by grammatical remarks on our use of the words. Summing up, although Wittgenstein’s ‘grammatical’ remarks on the actual use of concepts are intended as descriptions of what is *actually* going on in language use (hence they can be called empirical in this sense), to speak about “the empiricism of Wittgenstein’s later philosophy” without any qualification is seriously misleading.

I close this section with a point about the genealogy of the reading I’m advocating here. My proposal of thesis (3) was prompted up by Backer’s and Hacker’s (1992: 131) insight that, “perhaps” Wittgenstein’s point may be different from, and weaker than, their thesis (2). They suggest that Wittgenstein’s point in PI 65 – 67 is that the practice of explaining the word ‘game’ does not mention any essentialist definition of game<sup>23</sup>. In short, Wittgenstein’s only concern would be to highlight the fact that in the practice of using a word like ‘game’ speakers do not explain it in the way the essentialist may expect. I am, again, sympathetic with this sugges-

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sake of the fun or other satisfaction that is to be derived from participation in the activity and/or attainment of the objective”.

<sup>22</sup> The more general version of this proposition, “(Only) we know what is going on in our mind” is either a grammatical proposition, fixing (part of) the meaning of concepts like ‘knowledge’ or ‘mind’, or simply nonsensical, when viewed as a deep metaphysical truth (a piece of *a priori* knowledge). See PI, part II, p. 221e.

<sup>23</sup> Baker and Hacker (1992: 131) write: “Or perhaps [Wittgenstein needs] only [to] defend the still weaker thesis that the practice of explaining ‘game’ does not include singling out properties necessary for an activity to be a game”. As it stands, the final part of this statement is false, since Wittgenstein himself singles out what seems to be a property necessary to be a game, by calling games ‘proceedings’ (PI 66). Of course, since there are ‘proceedings’ which are not games, this is not a *sufficient* property to call something a ‘game’. For this remark see also H.-J Glock (1996: 121).

tion, even more than with thesis (2). However, if we ask *why* the practice of explaining ‘game’ does not mention any essentialist definition, any common feature, why speakers do not single out any such definition or feature, we can see that the answer to these questions is provided precisely by thesis (3): namely, because speakers do not know and do not need to know any such definition. The interpretation (3) is thus, I contend, more fundamental than their correct insight, in the sense that it is a thesis like (3) that can account of it. Speakers do not explain ‘game’ by giving the definition not because they are lazy or stupid, but because they do not know any definition. Thus, in my view, the Baker-Hacker insight is much more on the right track than their thesis (2).

### *Two arguments from textual consistency*

A good strategy to gain credit for the interpretation I propose here is to show that the objections Wittgenstein disputes with his imaginary interlocutor can be read as objections to the reading proposed by my interpretation. That is, given the reading I advance here, the interlocutor’s objections arise naturally. In this section I pursue this strategy and I discuss two such objections.

Interlocutor’s first objection runs as follows (in PI 69): even if we grant the point that speakers are not able to formulate a definition and explain ‘game’ by giving examples and by pointing to various resemblances, etc., it may not follow that they do not *know* that feature or definition. It might be that this essence is ineffable: it is “only other people whom we cannot tell exactly what a game is” while we do know what the essence of games is. Wittgenstein reconsiders this objection in PI 75; this objection challenges his assumption that speakers’ knowledge of what a game is is completely captured in the explanations they can offer.

I’ll address this objection below; before that, let me note that, according to PI 36, this move illustrates a way of proceeding highly characteristic to traditional metaphysics. The picture under whose spell we live indicates that *it must be* something (in this case, an essential feature) that we know and which *accompanies and supports* our use. However, when we question what we know when we use the word, we find nothing - that is, nothing physical, a common feature, to be captured in a definition. Then, because that picture holds us captive (PI 115) and dictates how we must see things, we postulate a *spirit*, something mental able to accompany and support the use of the word. The next step of this metaphysical *expla-*

*nation* is to suggest that we use this mental, spiritual, ineffable essence as a *guide* to our application of the word, despite the fact that we are not able to find a way to capture it in words, to make it *publicly* available.

In terms of how the text of PI flows, two substantial themes relevant for the first objection follow the family resemblance passages. First, we find Wittgenstein's analyses of what it means *to be guided* and second, his famous remarks about the impossibility of a private language. These points can be interpreted as addressing (not directly, but among other things) the above outlined objection. It does not serve my case here to delve into these two themes, but I count them as providing textual evidence that something like thesis (3) is what concerned in fact Wittgenstein in PI 65 - 67. This evidence is indirect in the sense that the acceptance of thesis (3) doesn't throw light on the difficulties posed by the celebrated 'argument' against the private language<sup>24</sup> or on the interpretative puzzles involved in the discussion about guidance<sup>25</sup>. My point concerns only the consistency of my reading with what follows in the *Investigations*. If it is true that thesis (3) captures Wittgenstein's main point in PI 65-67, then we can see that these discussions follow naturally. Reading the family resemblance point as I suggest here may not help understand what Wittgenstein says about guidance and privacy, but it gives us a promising clue as to why he thought he had to address these topics.

Now I examine whether my reading is consistent with the paragraphs PI 70 and 71, in which Wittgenstein challenges what is usually called Frege's 'ideal' of the determinacy of meaning (Glock, 1996). (Roughly, this is the view that any concept acts similarly to a mathematical function, sorting out things into two perfectly determined categories, those that fall under it, and those that don't.) Consequently, a concept lacking these 'sharp boundaries' is, in fact, not a concept at all. Wittgenstein takes up this second objection and, in PI 71, asks: "[I]s a blurred concept a concept at all?" Formulating it in analogy with the line of thinking proposed by the thesis (2), the Fregean ideal/dogma states that a concept can function only if it has sharp boundaries. It seems then that the PI 71 question asks how the lack of an exact definition of a concept affects its application – or, at least this is the reading thesis (2) suggests.

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<sup>24</sup> There is no such monolithic *argument*, in fact. See Canfield (2001) for a recent re-examination of the issue.

<sup>25</sup> See Wittgenstein's meticulous analysis of how we are guided by an arrow (PI 86), by somebody we are dancing with (PI 170, 172 - 190) or by a rule (PI 178) – for this last example, see Kripke's well-known (1982).

According to my interpretation, this is *not* what this question asks. This question should be read as asking how *speakers' lack of knowledge* of an exact definition affects their correct application of a concept. We are able to see this if we pay attention to the precise sense in which Wittgenstein challenges Frege's point. Very explicitly, he takes Frege's point to be that those concepts are rendered *unusable*: "This [the lack of sharp boundaries] presumably means that we cannot do anything with [them]" (PI 71). This remark is relevant since it shows that Wittgenstein does not start an investigation on the concepts themselves, as it were, but rather on speakers' use of them – that is, along the lines thesis (3) is developed. Moreover, in (BB: 19), in a passage ancestor to those in the *Investigations*, Wittgenstein says: "the actual usage...has no sharp boundary". Once again, the use of concepts is in question<sup>26</sup>, and not concepts themselves, so to speak. His concern with blurred concepts should then be understood in the following sense: how concepts blurred for speakers can have the use they have in speakers' linguistic practices? Concepts are blurred in the sense that it is speakers who do not have exact definitions for them; it is specifically this aspect makes the Fregean suspect we cannot use them.

The reasoning I'm pursuing here is similar to the one I advanced when I distinguished between theses (1) and (2) on one hand, and thesis (3) on the other. Wittgenstein cannot be taken to address the issue of 'blurred concepts' *simpliciter* (where 'blurred' means 'not having exact definitions'), since he did not (and cannot) prove that definitions do not exist. Given that he could draw no conclusion about the very existence of essentialist definitions, it is unreasonable to think that he is developing his thoughts by assuming this conclusion and asking: "How can speakers use the word 'game' correctly if (as we showed) there is no definition of games". This is so because he did not show, in fact, that there are no definitions. All that his descriptive method was able to accomplish was to make us realize that we do not know any definition. Therefore, I contend the correct interpretation of what is asked here is along the following line: "How can speakers use the word 'game' correctly, if (as we saw) speakers know no (and need not know) definition of games?"

Given Frege's view of language as calculus and thesis (3) (that speakers do not need to know exact definitions to use words *correctly*), the question 'how is this possible?' crops up naturally: in any calculation problem the emphasis naturally falls on the correctness of what people do. So,

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<sup>26</sup> O. Hanfling pointed out to me (personal communication) that 'usage' might not be interchangeable with 'use'. However, I assume they are synonyms.

the second question-objection (“[I]s a blurred concept a concept at all?”) asks whether or not we can do anything with these concepts. If we cannot identify any ‘sharp boundaries’, then, the query is, how do we distinguish between correct and incorrect uses of words, how do we *justify* our application of concepts? Although we saw that those concepts are not rendered unusable (by inspecting the practice of using them), the confusion still persists: how is any *successful* use possible if we do not master exact definitions of (some) concepts? As it is known, from here Wittgenstein goes on by analyzing the very idea of exactness, and, more generally, the assumption that natural language can be assimilated to a system of calculus. However, following Wittgenstein’s answers on these topics is beyond the scope of this paper.

### *Conclusion*

In my reading, Wittgenstein’s main target in the family resemblance passages is not the straightforward essentialist thesis ‘there is an essence of games (captured in the analytical definition)’, but, specifically, a view like ‘speakers need to know a definition / essence in order to apply the term correctly’. My reading of these passages is along the lines of thesis (3), and it is meant to dismiss this later view. Descriptions of the use of language show that speakers do not know any definition, any essence of games when they apply the term ‘game’ correctly. Therefore, no knowledge of such essentialist definition is necessary for the correct application of a word. The intended effect of thesis (3) on the nucleus of traditional essentialism is not rejection, but, so to speak, dissolution. Essentialism’s supposed foundational force should be neutralized, since essentialist definitions do not have any function in our use of a natural language concept. As Wittgenstein used to say, they are like cogs disconnected from the mechanism.

In light of this reading, Wittgenstein’s famous view on the intended effects of his philosophical method should look less dogmatic. We begin to understand how and why the philosophical problems associated with (definitional) essentialism should “*completely disappear*” (PI 133). The conclusion regarding the dissolution of the philosophical force of essentialism bears directly on what is usually taken to be the relevance and the aim of traditional metaphysics, to provide us with *foundational* results, with discoveries about the very nature of reality. When we recognize that our use of language is independent of what such enterprise may unearth (if anything), the relevance the metaphysician invokes for her inquiry into the na-

ture of things vanishes. For Wittgenstein, philosophers' claim to provide conceptual foundations (in the sense of supplying foundational justifications for our use of concepts) is simply an illusion. Good philosophy leaves everything as it is, bad philosophy strives for foundations. When these foundations are believed to be found, the immediate consequence is that (bad) philosophy proposes linguistic reforms, thus interfering with the actual usage of natural language concepts (PI 124).

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CARLOS DUFOUR

## Identity and Predication

### Observations on P. Monaghan's Thesis

Identity and predication are basic philosophical concepts which have often led to amazing considerations from the paradox of Antisthenes onwards to some of Hegel's philosophemes on judgement. Recently P. Monaghan pleaded in *Metaphysica* for an assimilation of identity and "property possession".<sup>1</sup> But property possession, in contrast to identity, is neither reflexive, nor symmetrical, nor transitive. What should we think about such an assimilation? Although Monaghan takes into consideration two objections to his thesis, there are other evident difficulties which cry for attention. Moreover his replies to these two objections are hardly comprehensible and, in my opinion, the use of the traditional concept of an entity's nature as well as the application of the fashionable concept of mereology do not make things clearer either. Finally, both of Monaghan's puzzles are not cogent enough to confirm his thesis.

#### *The main question and its difficulties*

Following Monaghan we can use global variables.<sup>2</sup> The application of a property to a logical subject can be reproduced by a specific relation as ":: $x :: u$ ". The scheme " $x :: u$ " means that  $x$  possesses the property  $u$ . It is true that there is a strong overlapping of property possession (also called "ontical predication") and identity, because in every adequate system it is a theorem that

$$(1) \quad \exists u (x :: u) \leftrightarrow x = x$$

That is, property possession and self-identity are equivalent. Nevertheless this is far different from

$$(2) \quad \exists x (x :: u) \leftrightarrow u = u$$

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<sup>1</sup> Monaghan 2005.

<sup>2</sup> See Bealer 1982: 76, 82; Mertz 1997: 207. Of course, if one works without logical types caution is needed to avoid the well-known paradoxes; in our limited context, however, we can be confident of having everything under control.

because it could happen that a property is not exemplified, although it goes without saying that it is identical with itself. While (1) is valid without restrictions, (2) requires that only exemplified properties are to be admitted. This means still a logical asymmetry between the left and the right side of predication which is by no means compatible with identity. The validity of (1) cannot mean that the relations of predication and identity are identical as in

$$(3) \quad \lambda x u (x :: u) = \lambda x y (x = y)$$

what Monaghan actually is maintaining. Because  $\forall x \exists u (x = u)$  is equivalent to  $\forall u \exists x (x = u)$  and every entity possesses a property would follow with (3) that every property, e.g. being a round square, should also be exemplified.<sup>3</sup> This difficulty is joined by many others which show that here the notions of property and predication are radically changed or that the assumption (3) is simply wrong.

- I) The exemplification of a property (e.g. having caught a cold) can be *de re* contingent, but not so identity. Therefore property possession and identity are different relations.
- II) As there are no things without properties, (3) implies that all entities are properties. But properties appear in contradictory pairs (*being round, being not-round*); common individuals like Socrates do not behave like this; therefore not everything can be a property.
- III) Every property would be self-applicable. The property *being a body* however is itself not a body. The property of non-existence should not exist and the negation of self-identity should be a property different from itself.
- IV) There could exist only one object. Suppose that  $a \neq b$ . Either both possess self-identity or for one of them, say  $a$ , should be valid  $a \neq a$ . The latter is absurd. If, however, both objects are possessing self-identity, i.e.  $a :: \lambda x (x = x)$  and  $b :: \lambda x (x = x)$ , follows with (3) that  $a = b$ .

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<sup>3</sup> The equivalence between  $\forall x \exists u (x = u)$  and  $\forall u \exists x (x = u)$  holds only if we have *global variables*, as above mentioned. In the case of a two-sorted language with  $x$  and  $u$  belonging to different sorts, there is no equivalence, because this would mean the same as an equivalence between „All  $X$  are  $U$ “ and „All  $U$  are  $X$ “.

- V) The negation becomes an enigma. Let us take two different but compatible properties  $u$  and  $v$ . From  $a :: u$  and  $\neg(a :: v)$  follows, because of Tertium Non Datur and complementation of the property  $v$ , that  $a :: \bar{v}$ , and thus  $u = \bar{v}$ . That is, no pair of properties could be compatible.
- VI) As Monaghan rightly observes, no property could be really universal, because from  $a :: u \wedge b :: u$  would follow with (3) that  $a = b$ .
- VII) As Monaghan mentions, an entity could possess only one property, because from  $a :: u \wedge a :: v$  would follow  $u = v$ .
- VIII) If relations are properties, states of affairs like  $1 < 2$  and  $2 < 3$  become inexplicable. The pair (1,2) should be identical with the pair (2,3) and in consequence  $1 = 3$ .

Perhaps one could avoid the one or the other difficulty. For example, in the logic and ontology of Mertz you can evade the objection (VII) because in it there are only particularized properties (instances) as predicates of objects. On the contrary, it does not seem possible to eliminate *all* the difficulties – they all arise from the questionable assimilation (3).

### *Replies and Elucidations*

It could turn out that the replies to (VI) and (VII) give a hint on the alternative conception of predication which Monaghan suggests.

As response to (VII) he tells us:

My response to this objection is that it is based upon a mistaken [conception ...] of the relation of property possession. Property possession is not a one-many relation that at least one entity can bear to many properties. Rather property possession is the one-one relation of identity.<sup>4</sup>

That would be a mere repetition of the thesis, if one did not add that the concept of the nature of an entity had to be introduced and an extensional mereology to be applied:

The nature of an entity is a property, which that entity possesses and which is complete in the sense that, for any property whatsoever, that property is a constituent part of the nature just in the case that property can be truly predicated of that entity (...) I understand the relations that ob-

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<sup>4</sup> Monaghan 2005: 73.

tain between an entity, its nature and the properties that are constituent parts of that nature to be the relation of extensional mereology.<sup>5</sup>

It is not easy to get illuminated by this explanation. If I interpret it correctly, the answer to the difficulty consists in the distinction between “true predication” (predication *lato sensu*) and “possession of a property” (predication *stricto sensu*) where the latter is a species and the first its genus. For a true predication it would be sufficient that a property is a proper or improper part of the subject’s nature, therefore sometimes without requiring identity between property and argument. Then the analysis of “true predication” should be:

$$(4) \quad F(a) : \equiv F \leq n(a)$$

where „ $n(a)$ “ stand for the nature of the entity  $a$  and „ $\leq$ “ for the relation of proper or improper part. If it is not so, Monaghan would not have shown that the possession of a property (in the usual sense) coincides with the relation of identity, but only that he prefers an alternative use of language. If one wants to go beyond liberty of stipulation and beyond verbal questions, one has necessarily to think a little bit about clearness and adequacy of the claim (4).

First of all, it is striking that the effort made by introducing the nature  $n(a)$  of  $a$  is superfluous, because  $a$  is just possessing the nature  $n(a)$  as a property („the nature of an entity is a property, which that entity possesses“, as Monaghan says). Then, because of  $a :: n(a) \rightarrow a = n(a)$ , one could simply explain the predication  $F(a)$  as  $F \leq a$ . Unfortunately, the concept of *part* is much less clear than the concept of predication.

Secondly, it becomes obvious that this analysis is not general enough to give an account of the predication of relations. If the book  $b$  is lying on the book  $a$ , should then the relation  $L$  of lying be a part of the nature of  $a$  (that is, simply, a part of  $a$ ) or of  $b$  or of what else? Because *being part of something* is also a relation, other doubts must arise here.

As already mentioned under item (I), the contingent predications represent an obstacle, because the “traditional” notion of nature is opposed traditionally also to the accidental features.<sup>6</sup> Perhaps *rational* is somehow part of the nature of Socrates, but by no means *having caught a cold*.

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<sup>5</sup> Monaghan 2005: 74.

<sup>6</sup> Here I can also refer to Gracia 1988: 2–3, 9–10, 118, 121. “What is common to the thing and other actual or possible things is usually referred to by philosophers who use traditional terminology as the *thing’s nature* (...) The features that a thing may or may not have, and thus are not necessary conditions for its kind of existence, are

Especially awkward is the effect of the obscure concept of *part*. Because the nature of Socrates – according to (3) – is supposed to be identical with Socrates, a part of his nature should be the same as a part of him. An eye is a part of Socrates but one cannot figure out how this eye should yield a true predicate of Socrates, a consequence which is implied by (4).

Moreover, properties permit predications (*lato sensu*) of other properties, too. Thus *human being* satisfies the property of *being exemplified by Plato*. Although we can maintain, or at least make an acceptable sense thereof, that *animal* is part of the property *human* – how can one possibly understand that *being exemplified by Plato* is also a part of *human*? Here we get into a dilemma. If *being exemplified by Plato* is not a part of *human*, because of (4), we cannot predicate *human* of Plato. If however the mentioned property is really part of *human*, because Socrates is a human being and the mereological relation  $\leq$  is transitive, follows with (4): Socrates is exemplified by Plato. Both consequences are absurd.

Summarizing: all these problems and many others arise if one defends thesis (3) by a distinction between predication *lato sensu* and *stricto sensu* and eventually applies (4) as elucidation.

#### *The motivation for the assimilation and the theoretical context*

So far we have explained succinctly a few reasons against the assimilation of predication and identity. But can there be also mentioned reasons in favour?

Let us disregard thesis (3) and return to the common conception of property possession. Monaghan perceives two puzzles in

$$(5) \quad x :: y \wedge \neg(y :: y)$$

if (3) is rejected. The “problem of relevance“ is presented this way (where *y* is taken as “red”):

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usually called *accidental*” (p. 2–3). Gracia seems to consider here nature as *universal*, i. e. as opposite to individual. It is not his intention to deny the traditional distinction between the nature of an individual and the nature of its accidents. The locus classicus for this is Aristotle, *Metaph.*, Z, c.4 1029 1 – 1030a 7, cf. Dufour 2005: 281–287.

For I ask: will  $x$  still be red, even if it does not bear the relation [of property possession] to  $y$ , but instead to some other non-red entity? And if not, why not?<sup>7</sup>

But it is always valid that  $x :: y$  or  $\neg(x :: y)$ . Therefore, if  $x$  does not possess the property  $y$ , then  $\neg(x :: y)$ . The question whether  $x$  bears the relation of property possession to  $y$ , i.e.  $x :: y$ , in case of  $\neg(x :: y)$ , can have only one answer: not at all. Why not at all? Because of the Contradiction Principle. Consequently, the “problem of relevance” does not yield a satisfying motivation for (3).

The so-called “problem of contribution” is presented this way (where  $y$  is taken again as “red”):

It is the problem of explaining how the non-red  $y$  *makes*  $x$  red. In other words, it is the problem of explaining how the non-red  $y$  contributes redness to  $x$ . And it is a problem that seems wholly mysterious to me.<sup>8</sup>

But the problem arises only if one engages oneself to two questionable assumptions:

- i) *Quasi-Causality*. If  $x :: y$ , the  $y$  itself has to make somehow that  $x$  exemplifies the property  $y$ .
- ii) *Homogeneity*. If something makes that  $x :: y$ , then it has also to be a  $y$ .

But which insights do yield us evidence for all that? If  $x$  exemplifies a property  $y$ , the reason or cause thereof needs not to be  $y$  itself. The reason for the application of a universal property can be another general property, or an individual accident, or an instance of Mertz, or an external fact.

Suppose that Socrates is short-sighted. The reason thereof (if there is any such reason) may be that his eyeballs have a certain shape. It would be odd to demand this shape also being short-sighted. So far, only these presuppositions (i) – (ii) are responsible for the problem but not the concept of predication in the usual meaning.

Perhaps one could understand Monaghan’s thesis as a hint to Bundle-Theories. According to them a function can be defined which maps every individual into the set of its properties (instead of a set you can choose a conjunction of properties, an ontological totality or your favourite form of collecting entities). In a second step one postulates the identity between

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<sup>7</sup> Monaghan 2005: 72.

<sup>8</sup> Ibid.

the individual and its set of properties. So every predication which does not involve identity is equivalent to the fact that the predicated property is an element of this set of properties – if elements are “parts”, then every property of the individual is part of the “nature of the individual”.

In this case many questions are left open, depending on how the Bundle-Theory is shaped. Normally, the point would consist in the inverse function which constitutes an individual from every set of properties. This allows us to enjoy Meinongian objects. It is quite rare that someone wants to defend an Aristotelian approach and ends up explaining universals as parts of individuals.<sup>9</sup> Of course it is true that thus there are no universal properties without individuals, but in return other absurdities are emerging.

I suppose that

- (i) an Aristotelian view has to avoid both Bundle-Theories and Bare Particulars.
- (ii) anyway, a *bridging-principle* must be adopted in order to connect predication and possession of universal parts.

That is:

(BP)  $F(a)$  iff  $F$  is part of  $a$

what equals the definition (4). But now there is a problem if we take extensional mereology seriously. Because of the mereological theorem<sup>10</sup> of Strong Supplementation: if Socrates has the universal  $U$  as a proper part, the entity  $s$ -without- $U$  must exist, i.e.:

(6)  $\exists x(x \neq s \wedge x = s - U)$

But we can also remove (via mereological sums and eventually supplementation) *all* universal parts, that is, we should get the equation:

(7)  $((s - U) - U^*) - U^{**} \dots) = x$

What about this  $x$ ? If there is no such entity  $x$ , we get Bundle-Theory (Socrates is just a sum of universals). But if  $x$  exists, we obtain – because of the bridging-principle (BP) – just Bare Particulars, little *Dinge an sich*.

Perhaps one could weaken (BP) but such a solution would look too much ad hoc. A revision of the mereology for universal constituents is still open, but it is not clear how to tackle this problem. The question of the forms of

<sup>9</sup> There are, however, remarkable exceptions, like the two *Laws of Immanent Realisms* in Smith 1997: 106, 119.

<sup>10</sup> See Simons 1987: 29.

predication however can only be explained in this theoretical context. If we take the question isolated all evidence points against the assimilation of predication and identity.

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URIAH KRIEGEL

## Tropes and Facts

### INTRODUCTION/ABSTRACT

The notion that there is a single type of entity in terms of which the whole world can be described has fallen out of favor in recent Ontology. There are only two serious exceptions to this. Factualists (Skyrms 1981, Armstrong 1997) hold that the world can be fully described in terms of facts. Trope theorists (Williams 1953, Campbell 1981, 1990) hold that it can be fully described in terms of tropes. Yet the relationship between facts and tropes remains obscure in both camps' writings. In this note, a distinction between (the names of) events and facts, due to Vendler and Bennett, is extended to distinguish between (the names of) tropes and facts. On its basis, a portrait of the domain of abstract particulars is sketched. The purpose is to contribute to our understanding of both forms of (if you will) metaphysical monism by offering a principled distinction between them.

### *1. Events and Facts*

Jonathan Bennett (1988), following Zeno Vendler (1967), distinguishes between events and facts. Consider the indicative sentence

(1) I strolled in the park.

(1) is a sentence, not a name. So it does not name anything, indeed any *thing*. But there is a standard way to produce names from sentences – nominalization. One way to nominalize (1) is with the *perfect nominal*

(2) My stroll in the park

Another way is with the *imperfect nominal*

(3) My strolling in the park

(2) is called a ‘perfect’ nominal, because the nominalization leaves no trace of a verb. In (3), by contrast, there is a trace of a verb, so it is said to be an ‘imperfect’ nominal.

Both (2) and (3) are not sentences but names. For they can be plugged into the subject position in a subject-predicate sentence, as in

(4) My stroll in the park was noted by the neighbor.

(5) My strolling in the park was noted by the neighbor.

Here ‘noted by the neighbor’ predicates the name-bearers of ‘My stroll in the park’ and ‘My strolling in the park’. What are these name-bearers? According to Vendler and Bennett, the former names an *event*, whereas the latter names a *fact*.

In general, imperfect nominals are the names of facts.<sup>1</sup> Vendler and Bennett offer several arguments in favor of this thesis.<sup>2</sup> One basic reason to accept this thesis is that (3) can be transformed into the very straightforward nominal

(6) The fact of my strolling in the park

(3) and (6) are surely co-referential. There is no doubt that (6) names a fact. Therefore, (3) names a fact too. In general, imperfect nominals like (3) (which feature gerunds, e.g., ‘strolling’) are always interchangeable with some imperfect nominal similar to (6) (i.e., a nominal which features the operator ‘the fact of’).

The same is not the case with perfect nominals. Thus, (2) cannot be transformed into a similar straightforward nominal. For the following construction is ungrammatical:

(7) The fact of my stroll in the park

The only straightforward nominal (2) can be transformed into is

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<sup>1</sup> By “fact” we mean something like the traditional states of affairs. The term “fact” is used here because “state of affairs” is not a very ordinary term, but rather technical and theoretical. To the extent that we want to see how these sorts of entity are named in ordinary discourse, we would do better to use such an ordinary language term as “fact.”

<sup>2</sup> See, again, Vendler 1967 and Bennett 1988.

(8)The event of my stroll in the park

Again, (2) and (8) are co-referential, and given that (8) surely names an event, (2) names an event as well.

## 2. Tropes and Facts

Vendler and Bennett start out with indicatives featuring verbs; this is because they are interested in facts mainly in the context of their difference from events. But the same analysis can be applied to indicatives featuring the copula, such as

(9)The park is nice.

(9) has a perfect nominalization in

(10) The park's niceness

And an imperfect nominalization in

(11) The park's being nice

We may say that (10) is a 'perfect' nominal, in that there is no trace of the copula in it, whereas (11) is an 'imperfect' nominal, since there *is* a trace of the copula in it. Both can be used as names in a subject-predicate sentence:

(12) The park's niceness was noted by the neighbor.

(13) The park's being nice was noted by the neighbor.

The suggestion I would like to make is that (10) is the name of a trope, whereas (11) is the name of a fact.<sup>3,4</sup>

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<sup>3</sup> We can accept this claim regardless of our take on the more general pretensions of trope theory.

<sup>4</sup> Tropes have been first introduced into the modern literature, under that name, by Williams (1953). But by different name, they can be found already in Stout (1923),

We can see this by noting, again, that (11) can be transformed into the straightforward nominal

(14) The fact of the park's being nice

(14) is not eloquent, but it's English. The following is not, however:

(15) The fact of the park's niceness

This is because the name-bearers of perfect nominals (whose parent sentences are true) are tropes, not facts. Facts are the name-bearers of imperfect nominals (whose parent sentences are true). The argument is the same as in the case of nominals featuring verbs: Since (11) and (14) are co-referential, they name the same thing, and given that (14) names a fact, (11) must name a fact as well.

This distinction between names of tropes and names of facts conforms with our intuitions. If niceness is a universal property, then the park's niceness is a particularized property, that is, a trope. But the park's being nice is not a property at all. (A property of what?) The park's being nice is just a fact. The difference can be brought out by comparing the following pair of sentences:

(16) The park's niceness was ignored by the neighbor.

(17) The park's being nice was ignored by the neighbor.

(16) and (17) are very different. In (16), what the neighbor is stated to ignore is *something about the park*, namely, its niceness. That is, what she ignores is a property of the park, albeit a particularized one. By contrast, in (17), what she is stated to ignore is not something about the park, but something altogether different: she ignores something about the way things are – *something about the world*. The park's being nice: the neighbor ignores that this is how things are. That is, she ignores a certain *fact*, a fact about the world.

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and some would claim that Aristotle's "individual accidents" are in effect tropes. Trope theory, which makes many ontological claims on behalf of tropes, has been developed mainly by Campbell (1981, 1990), with inspiration from Williams' original piece.

### 3. *The World of Abstract Particulars*

We have considered four kinds of nominal: (i) perfect nominals whose parent sentences feature the copula (e.g., “the park’s niceness”), (ii) perfect nominals whose parent sentences feature a verb (e.g., “the stroll in the park”), (iii) imperfect nominals whose parent sentences feature the copula (e.g., “the park’s being nice”), and (iv) imperfect nominals whose parent sentences feature a verb (e.g., “the strolling in the park”). The suggestion I have made is that (i) name tropes, (ii) name events, and (iii) and (iv) name facts.

One may hold that these four kinds of nominal refer to four different kinds of *abstract particulars*. Although this is problematic, let us for the purposes of present discussion say that a thing of kind K is abstract just in case there can be more than one K in the same place at the same time, and that a thing of kind K is particular just in case it cannot be in more than one place at the same time.<sup>5</sup> The trope of the park’s niceness is in the same place at the same time as the park’s vastness, but it cannot be in any other place at the same time. So the park’s niceness is both abstract and particular. All tropes are.

By this rough test for abstract particularity, tropes are not the only abstract particulars. Facts are too. There can be more than one fact occurring in the same place at the same time, but the same fact cannot occur in more than one place at a time.<sup>6</sup> Thus, the fact of the park’s being nice oc-

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<sup>5</sup> This characterization is not unproblematic. For starters, it does not allow us to say that numbers are abstract, since numbers have *no* spatial location. There are three ways to deal with this problem: (i) redefine abstractness as being either a-spatial or at the same place at the same time as other entities of the same kind; (ii) deny the existence of numbers (see Field 1980); (iii) claim that numbers do have spatial location, as some structural realists may do (see Maddy 1980). A second problem for the characterization is that it makes coincident objects – such as the statue and the clay – come out abstract, since they share the same spatial location at the same time. This is a difficulty indeed, and the only way I can see of dealing with it is to deny that the statue and the clay are two different things (as in Yablo 1987). A third problem is that it is not obvious how to individuate location, in a way that tells us definitely when  $L_1$  and  $L_2$  are one and the same location and when two distinct location. For all these reasons, we may do well to use the characterization in the text not as a definition of abstract particularity, but as a rough test (or indication) for abstract particularity.

<sup>6</sup> There might be a problem here with characterizing *disjunctive facts* as abstract and *conjunctive facts* as particular. A fact such as the park’s being nice or the zoo’s being impressive is neither where the park is nor where the zoo is, and therefore cannot be

curs at the same time in the same place as the fact of the park's being vast; and it occurs only where the park is and could not occur anywhere else at the same time. So there are (at least) two kinds of abstract particulars: tropes and facts.<sup>7</sup>

Several philosophers have argued, quite plausibly, that events are tropes.<sup>8</sup> If we treat events as a subgroup of tropes, it appears that perfect nominals are generally the names of tropes – either event tropes or non-event tropes – whereas imperfect nominals name facts.

Within the group of tropes, then, we have events (e.g., the stroll in the park) and non-event tropes (e.g., the park's niceness). Let us call the former *dynamic tropes* and the latter *static tropes*. We may draw a parallel distinction between *dynamic facts* and *static facts*. A dynamic fact (e.g., the strolling in the park) is named by an imperfect nominal whose parent sentence features a verb, whereas a static fact (e.g., the park's being nice) is named by an imperfect nominal whose parent sentence features the copula.

The emerging picture is of a structured domain of abstract particulars. Abstract particulars divide into two groups – tropes and facts – that subdivide into two subgroups, dynamic and static tropes and dynamic and

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co-located with any other fact. And a fact such as the park's being nice and the zoo's being impressive is both where the park is and where the zoo is, and is therefore in two different places at the same time. One response could be to reject the existence of disjunctive and conjunctive facts. A better response, though, would be to embrace such facts, but claim that they can be analyzed, in turn, in terms of atomic facts, or at least facts that are neither conjunctive nor disjunctive. A third option is to take this second line with respect to conjunctive facts and the first one with respect to disjunctive facts – in the same way some philosophers accept conjunctive properties but not disjunctive ones (e.g., Armstrong 1978).

<sup>7</sup> This is something that escaped much of the discussion of these sorts of entities. Many philosophers use “trope” and “abstract particular” interchangeably. But tropes are not abstract particulars *by definition*, even if they are *by necessity*. What tropes are *by definition* is particularized properties. Particularized properties happen to be abstract particulars, but it turns out that so do facts. If one defines tropes as abstract particulars – which I chose not to do – then of course tropes are the only abstract particulars; but there is still a distinction between particularized properties and facts, as two kinds of tropes.

<sup>8</sup> For the most comprehensive formulation and defense of this view, see Lombard 1986.

static facts. Each of the subgroups has its own proprietary nominal referring to it. This structured domain can thus be represented as follows:

Linguistic expressions	Perfect Nominal	Imperfect Nominal
Copula	<b>Static Trope</b>	<b>Static Fact</b>
Verb	<b>Dynamic Trope (Event)</b>	<b>Dynamic Fact</b>

The result is a cohesive account of the world of entities that are neither concrete particulars nor universals (abstract or concrete).

### *Conclusion*

Both tropes and facts have been offered by 20<sup>th</sup> century philosophers, mainly Australian, as the fundamental entities of the world: Williams (1953) and Campbell (1981, 1990) for tropes, for instance, and Skyrms (1981) and Armstrong (1997) for facts. Yet a clear distinction between these two kinds of entity is hard to come by. It would be somewhat *recherché* to claim that the distinction I have offered between the canonical ways of referring to facts and tropes might be useful in deciding which (if any) would be fit to serve as the bedrock of reality. My hope is not that the framework I have sketched *settles* such issues, but rather that it *illuminates* them.<sup>9</sup>

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<sup>9</sup> For comments on an earlier version of this paper, I would like to thank David Chalmers, Anthony Newman, Bernard Nickel, and Carolina Sartorio.

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## To Continue with Continuity

### ABSTRACT

The metaphysical concept of continuity is important, not least because physical continua are not known to be impossible. While it is standard to model them with a mathematical continuum based upon set-theoretical intuitions, this essay considers, as a contribution to the debate about the adequacy of those intuitions, the neglected intuition that dividing the length of a line by the length of an individual point should yield the line's cardinality. The algebraic properties of that cardinal number are derived pre-theoretically from the obvious properties of a line of points, whence it becomes clear that such a number would cohere surprisingly well with our elementary number systems.

### *1. Introduction.*

Were there physical continua, e.g. space-time, there would be an objective fact of the matter about the truth of our hypotheses about continuity. One hypothesis that has shaped modern mathematics (and thence logic and metaphysics) to a very great extent is that the geometrical line and the real number line are isomorphic, which I will call C-D, as it is due to Cantor and Dedekind. That hypothesis is assumed by almost all scientists nowadays, but nonetheless its philosophical analysis might one day benefit from our having available the widest possible range of alternative hypotheses (e.g. see Ehrlich 1994, not to mention such category-theoretic possibilities as synthetic differential geometry). In this essay I take an informal (pre-theoretic) look at one neglected hypothesis. I cannot consider any of its philosophical ramifications in any depth, not as well as defining it (in §2) and exhibiting its structural coherence (in succeeding sections), but I will at least be enabling that to be done.

My hypothesis may be introduced as an extrapolation from the familiarly finite. If we consider sand grains to be cubic millimetres of silicate, to keep things simple, then a sandstone mountain, say  $M$ , composed entirely of such grains and occupying a cubic kilometre, would contain  $10^9 \text{ m}^3 \div 10^{-9} \text{ m}^3 = 10^{18}$  grains. That may be expressed, using Kessler's (1980, 69) empiricistic notation, as  $10^{18}(M, \text{being-a-grain})$ . By extrapolating, it is not

hard to imagine that, if lines were composed of points, each of length 0, then a line of arbitrary unit length  $u$  might contain  $1 \cdot u \div 0 \cdot u = 1/0$  points. Such extrapolations, from finite cases to an infinite case, are unreliable, but they do not necessarily fail (e.g. they found set theory, see principle  $b$  of Hallett 1984, p. 7) and that one turns out to be coherent enough, as you will see.

First I should define my terms a little more precisely. Let the *primitive* line be the line that would be *physically* instantiated, e.g. as time, were time infinitely divisible. That idea, of a pre-theoretic geometrical line, makes sense whether or not there actually are any physical continua, and is more fundamental (conceptually) than such formal mathematical models of it as the standard real number line. Let primitive *cardinality* be what collections that may be related by *bijections* (one-to-one correlations) must have in common. That concept is also fundamental, and therefore its nature is also highly debatable, but the following may contribute to such debates, so I shall simply assume that definition. Finally, let  $k$  denote the (primitive) cardinality of the continuum. In other words, if lines were composed of points, and if there were physical continua, so that a line of points, say  $L$ , would be instantiated, then we would have  $k(L, \text{being-a-point})$  in Kessler's notation.

I will begin to clarify what the possibility of  $k$  resembling  $1/0$  amounts to in §2, but an immediate problem is that you may already regard  $1/0$  as an impossible whole number. You may think, for example, that from  $1 \times 0 = 2 \times 0$  we would be able to deduce  $1 = 2$ , were we to allow arithmetical division by 0, so I shall end this introductory section by challenging that particular reason. For an apposite historical analogy, when Cantor introduced (informally) his transfinite whole numbers he first had to challenge prejudices against their possibility (see Cantor 1883, pp. 892-893) before arguing that they were not just possible, but were actually coherent and useful.

Now, although the  $1 = 2$  above does follow from assuming  $0/0 = 1$  (together with associativity), why should  $0/0 = 1$ ? One reason might be that  $a/a = 1$  whenever  $a/a$  is defined at present (for finite  $a$ ), and additionally defining  $0/0$  is like allowing  $a$  to be 0. But that kind of extrapolation is notoriously unreliable, and note that indeterminate forms within the calculus are often denoted by  $0/0$ , the reason being that all the finite numbers yield 0 upon multiplication by 0. In fact, it need only follow, from dividing  $1 \times 0 = 2 \times 0$  by 0, that  $0/0$  includes  $x$  *iff* (if and only if) it contains  $2 \cdot x$ , and that would allow arithmetical division by 0 (if not as a function) if  $0/0$  could be

a *collection* of numbers (if not a set of numbers). I consider such collections in §2; but incidentally, multifunctions and mereological collections have indeed proved to be mathematically coherent and useful.

Another reason might be that ‘division by  $x$ ’ means ‘multiplication by the multiplicative inverse of  $x$ ’ within the number fields, and  $x$ ’s multiplicative inverse is whatever yields 1 when multiplied by  $x$ . But from  $0/0 \neq 1$  it need only follow that division by 0 is not allowed within number fields. You will see (in §5) that it is allowed within number pitches, which contain number fields in an algebraically strong way. Note that I will not be suggesting that functions and fields are not useful. They are extremely useful, but we may certainly extend that repertoire so as to include other structures that nature *might* be instantiating. By analogy, there is an obvious utility to having the cardinality of a collection increase by 1 when a new object is added to it, yet we may consider infinite cardinals whenever we have reason to.

## 2. *A Non-set-theoretic Cardinal.*

We may begin to consider the metaphysical possibility of  $k$  resembling  $1/0$  by considering the coherence of adjoining an undefined symbol  $\#$  to the natural numbers (in §3), where the informal properties of  $\#$  are derived from two heuristic assumptions:

( $ha_1$ ) that  $\#$  is a possibility for  $k$ ; and

( $ha_2$ ) that  $1/\#$  is the length of a point.

Such an approach is relatively direct because it is analytically metaphysical (i.e. pre-theoretic, not unlike Cantor 1883) rather than axiomatically mathematical (usually set-theoretic, cf. Kitcher 1983, p. 190) and so it avoids prejudging what kinds of numbers are possible (cf. §7). Note that  $\#$  is not defined to be  $1/0$ , the reason being that  $\#$  is, if coherent, a cardinal number, which is a more fundamental kind of number than a ratio of magnitudes.

In case, at the outset, your suspicions are roused by  $\#$  not being one of our numbers already, note that  $\#$  cannot be a set-theoretic cardinal: were  $k$  the cardinality of a set,  $0 \cdot k$  would equal 0, because the Cartesian product of  $\emptyset$  (the empty set) with any set is  $\emptyset$  (note that, for an arbitrary number  $x$ ,  $0 \cdot x$  does not necessarily have to equal 0, e.g. it need not within category theory). Furthermore, the major alternative to set theory as a foundation for mathematics has been constructivism, which prefers its lines *not* to be full of points. Consequently, even a coherent  $\#$  may well have been over-

looked. You will be better placed to decide whether or not the metaphysical (and perhaps empirical) hypothesis  $k = \#$  coheres with the concept of a line of points after the following essay. But of course, it is hardly unnatural to treat continua mereologically, whether or not they are full of points. And in order to presume as little as possible about what numbers really are, in this pre-theoretic essay, I shall require their sort of collection to be an informal kind of atomic mereological sum (as formally described in Simons 1987, p. 14). A suitable collection has the following 4 informal properties (at least) and I call such a collection a *mere-sum* and denote it by square brackets.

The *first* property of mere-sums of numbers is that the individual numbers are regarded as atoms. That is, mere-sums of numbers are not also mere-sums of whatever comprises those numbers (in a different way) if anything does (e.g. their elements, if numbers are classes). *Secondly*, because the mereological sum of  $x$  and  $y$  is just  $x$  and  $y$ , internal brackets can be eliminated (e.g.  $[[1, 2], 3] = [1, 2, 3]$ ), and the mere-sum of a single number is merely that number (e.g.  $[1] = 1$ ). *Thirdly*, two mere-sums are naturally defined to be equal *iff* a bijection between them may relate each atom with an equal atom. Consequently,  $[x, y] = y$  *iff* each atom of  $x$  is also an atom of  $y$ , so that  $x$  is a part of  $y$  (formal mereologies being part-whole theories), which is abbreviated to  $x \leq y$  below (e.g.  $[1, [1, 2]] = [1, 2]$ , so  $1 \leq [1, 2]$ ). Furthermore, if  $x \leq y$ , and also  $z \leq x$  implies  $x \leq z$ , then  $x$  is an atom of  $y$ , abbreviated to  $x @ y$  below (e.g.  $1 @ [1, 2]$ ). And *finally*, arithmetical operations naturally distribute over mere-sums of numbers (e.g. adding 1 to both 1 and 2 yields 2 and 3, and so  $1 + [1, 2] = [(1 + 1), (1 + 2)] = [2, 3]$ ).

Mere-sums make very natural pre-theoretic collections (of numbers) and not even that much geometrical mereology will be required, not explicitly (cf. §7). But  $\#$  does require that primitive lines might be made of points. Points are quite possible and conceivable, of course; e.g., an imaginary black square on a white background has points at its corners, where its edges intersect. Although planes do seem more like, for example, glass panes than sandpaper, that intuition cannot imply that they are not full of points, because points, having size 0, are infinitely smaller than sand grains, which hardly conflicts with planes being infinitely smoother than sandpaper. Furthermore, lines in planes are not like scratches put *onto* glass panes, because the *positions* of such scratches would make much better analogies for primitive lines, and they were clearly there already. In Aristotle's (spatial) line, a point had a potential existence that was actualised

only if something happened there, but it is not uncommon to think that if something *could* happen there then an actual position (or point) must have been there already, so that it could happen *there*. While points are not a metaphysical necessity (see Dummett 2000 and Slater 2003), if points do exist then, as there is nowhere in a line where it cannot be intersected by another (0-width) line, lines are clearly *full* of points. A primitive line is effectively thought of as being some infinitude of points with the single thought that such intersections may occur anywhere within it. So the fact that it is also a single operation that yields  $1/0 = \#$  (see §3) indicates that  $k = \#$  is not an intrinsically unreasonable hypothesis.

Nor is  $k = 2^{\aleph_0}$  of course, where  $^$  denotes standard cardinal exponentiation (because I use the more familiar superscript notation for a more familiar form of exponentiation in §4) and  $\aleph_0$  is the cardinality of the natural numbers (regarded as an actual infinity). That equation for  $k$  follows from C-D, so note that the significance of that widespread assumption may be assessed properly only if all the alternatives to it are also considered. For almost a hundred years, mainstream mathematicians have been using numbers that are (isomorphic to) ZF sets, not least because geometry was reduced to analysis following Descartes, and analysis was reduced to set theory following Cantor. But while ZF set theory provides mathematics with a *definite* subject-matter, not incoherently (e.g. see Steinhart 2002), it is hardly a *comprehensive* theory of cardinality, such as would be required for deciding the metaphysical propriety of  $\#$ . E.g., we ourselves instantiate *the* natural numbers, so we can hardly just define them (see Hamming 1998), and note that it is only *their* emulation by some ZF sets (the finite von Neumann ordinals) that justifies those sets being called natural numbers within ZF. (For various thoughts about *the* natural numbers, see Tieszen 1989, Dehaene 1997 and Heck 2000.) In particular, the totality of the natural numbers might even be potentially infinite, for all we really know. That possibility is already reflected within mathematics by the persistence of constructivism, and it will be accommodated in the last few sections of this relatively platonistic essay by considering lines of  $\#$  points in two cases, C-I, in which the natural numbers form an actual infinitude, and C-II, in which they do not. At the opposite extreme, for another example, we know that the cardinality of the totality of all the sets, say  $\Omega$ , is not the size of a set, and yet it is certainly a *primitive* cardinal number because  $\aleph_\alpha$  denotes a transfinite cardinal *iff*  $\alpha$  denotes an ordinal (so those two classes correlate one-to-one). (Furthermore, it is obviously coherent to regard the totality of the cardinals and the totality of the ordinals as 2 totalities, de-

spite the impossibility of doing so within ZF.)

### 3. From Notion to Fraction.

First I adjoin  $\#$  to the *natural* numbers,  $N =_{\text{df}} [1, 2, 3, \dots]$ , to make  $N^\# =_{\text{df}} [N, \#]$ , which I call the *notional* numbers, in order to see how strong the arithmetic of  $N^\#$  can be, given  $ha_1$  and  $ha_2$ . That is a good beginning (especially if C-II is the case) because little is as straightforward as the *informal* arithmetic of the natural numbers. (Incidentally, although a formal extension of an algebraic structure would define new operations upon new objects, with some part of the new structure being isomorphic to the whole of the old domain, I shall call that part and the new operations by their old names as far as possible, for clarity.) The exclusion of 0 from  $N$  might strike you as odd, because we are used to including it in our ZF set of natural numbers, but it is desirable to exclude it here because the informal properties of  $\#$  are going to be obtained via the concept of a *line* of  $\#$  points. In that context, 0 is primarily the length of a point, it is a magnitude (an answer to ‘How much?’) rather than a multitude (an answer to ‘How many?’), and so 0 will be introduced as an abbreviation for  $1/\#$  when I consider the ratios of notional numbers, at the end of this section. Of course, 0 is indeed a finite cardinal number (if one of a unique kind) and so beginning with  $[N^\#, 0]$  would have been a coherent (if less clear) alternative.

To begin with (where  $n$  is, as usual, a natural number variable)  $\# + n$ ,  $\# + \#$ ,  $\# \cdot n$  and  $\# \cdot \#$  all equal  $\#$ , for the following reasons. The first equation,  $\# + n = \#$ , could hardly be false given the second, which is a special case ( $n = 2$ ) of the third,  $\# \cdot n = \#$ , which follows (via  $ha_1$ ) from how the points of a line of length  $n$  correlate one-to-one with the points of a unit line. Replacing  $\#$  by  $k$  in the fourth equation makes it say that planes have the same cardinality as lines, as we would expect nowadays (and it also follows from  $ha_2$  below). All 4 equations should be unsurprising nowadays, as they remain valid if any transfinite cardinal replaces  $\#$ . And, as we would expect of arithmetical operations that may apply to numbers of points, addition and multiplication may both remain associative and commutative (the consistency of retaining those algebraic strengths is clear enough because any finite expression containing  $\#$  just equals  $\#$ ). Furthermore, it is trivial to check that multiplication distributes over addition, via a few typical equations such as  $\# \cdot (\# + n) = \# \cdot \# = \# = \# + \# = \# \cdot \# + \# \cdot n$ .

Note that  $N^\#$  is clearly (just as  $N$  is) closed under both operations. I

call a mere-sum  $S$  *closed* under a (binary) commutative operation  $\circ$  if  $x \circ y @ S$  whenever  $x$  and  $y$  are atoms of  $S$  because that is isomorphic to the familiar definition of closure (a set  $\mathbf{S}$  being closed under  $\circ$  if  $x \circ y \in \mathbf{S}$  whenever  $x$  and  $y$  are elements of  $\mathbf{S}$ ). Isomorphic definitions will not usually be stated explicitly, for brevity, but that definition extends rather naturally to a concept that is more useful with the inverse operations. I say that  $S$  is *mere-closed* under  $\circ$  if  $x \circ y \leq S$  whenever  $x$  and  $y$  are atoms (or other parts) of  $S$ . That concept coheres with the informal meaning of algebraic closure because if a mere-sum is mere-closed then operating within it cannot generate anything that is not there already.

The *inverse* of  $\circ$  is usually an operation  $\mathbf{i}$  such that  $x \mathbf{i} y = z$  iff  $x = y \circ z$ , e.g.  $3 - 2 = 1$  because  $3 = 2 + 1$  and nothing else (of current interest) yields 3 when 2 is added to it. But  $\# - \#$  and  $\#/\#$  (which is  $0/0$ ) will be collections of numbers (cf. §1), so a more appropriate definition of  $\mathbf{i}$  (in terms of  $\circ$ , and within a domain containing atoms  $x$ ,  $y$  and  $z$ , and which I call  $In$ ) is  $z @ x \mathbf{i} y$  iff  $x @ y \circ z$  (which includes the usual definition as a special case). E.g.,  $\# - n = \#$  follows from  $In$ , since  $\# + n = \#$  and  $N$  is closed under addition, and  $\#/n = \#$  also follows from  $In$ , since  $\# \cdot n = \#$  and  $N$  is closed under multiplication. Similarly,  $\# - \# = N^\#$ , since  $\# + \# = \# = n + \#$ , and  $\#/\# = N^\#$ , since  $\# \cdot \# = \# = n \cdot \#$ . Subtraction and division are not closed in  $N$ , so they are not mere-closed in  $N^\#$ , and unsurprisingly they are neither associative nor commutative.

For our first surprise, however, multiplication cannot distribute over subtraction, within  $N^\#$ , because  $(2 - 1) \cdot \# = \#$  does not equal  $2 \cdot \# - \# = N^\#$ . That must seem like bad news for  $\#$ , but consider the (informal) set  $\mathbf{N} \cup \{0, \aleph_0\}$ , where  $\mathbf{N}$  is given by  $n \in \mathbf{N}$  iff  $n @ N$ . Cardinal multiplication cannot distribute over subtraction within that set, lest  $\aleph_0 = (2 - 1) \cdot \aleph_0 = 2 \cdot \aleph_0 - \aleph_0 = \aleph_0 - \aleph_0 = (1 - 1) \cdot \aleph_0 = 0 \cdot \aleph_0 = 0$ . Defining  $\aleph_0 - \aleph_0$  would be useful, e.g. removing  $\aleph_0$  objects from  $\aleph_0$  objects would leave  $m$  objects, where  $m \in \mathbf{N} \cup \{0, \aleph_0\}$ , but  $N^\#$  would be relatively strong anyway, even were  $\aleph_0 - \aleph_0$  undefined, because at least  $\# - \#$  is defined. So it is likely that failures of distributivity are just as natural for infinite cardinals as failures of commutativity are for infinite ordinals (cf. my glance at exponentiation in §4). And although multiplication will stop distributing over *addition* when negative numbers are adjoined (in §4), it is not especially unnatural for a commutative multiplication to fail to distribute over a commutative addition (e.g. it may do so within category theory).

Anyway, consider next the *ratios* of the notional numbers, because within the motivating context of a line of  $\#$  points we may consider  $\#$

points, and  $n$  points for any  $n$ , and also  $n$  line intervals. The continuity of the line makes it possible (in principle) to continue to subdivide intervals endlessly, so it is natural to extend  $N^\#$  next to a domain that is mere-closed under division. I shall call a ratio of two notional numbers, if it is not  $\#/\#$ , a *fractional* number, an atom of  $F^\#$ . The elementary arithmetic of  $F^\#$  subsumes that of  $N^\# \subseteq F^\#$  of course, and includes that of  $1/\#$ , and the remaining atoms of  $F^\#$  are of the form  $r = n/m$ , where  $n$  and  $m$  are relatively prime natural numbers with  $m > 1$ . Addition and multiplication may remain *commutative* and *associative*, with multiplication *distributing* over addition (it is trivial, if tedious, to show the consistency of retaining those algebraic strengths). Note that dividing the notional equation  $\# \cdot n = \#$  by  $m$  yields  $\# \cdot r = \#$  (since  $\#/m = \#$ ), while dividing  $\# \pm n = \#$  by  $m$  yields  $\# \pm r = \#$ , and multiplying  $\#/n = \#$  by  $m$  yields  $\#/r = \#$ .

It follows from  $ha_2$  that  $1/\#$  is the additive identity because, for example, ignoring one of the end-points of a line interval would not affect its length, so  $n \pm 1/\# = n$  and  $r \pm 1/\# = r$ , from which  $\# \pm 1/\# = \#$  follows by adding  $\#$ . And via  $In$ ,  $n - n = 1/\#$  and  $r - r = 1/\#$ . Consequently,  $1/\# + 1/\# = 2 \cdot n - 2 \cdot n = 1/\#$ , and  $r/\# = r^2 - r^2 = 1/\#$ , and  $(1/\#) \cdot (1/\#) = 2 \cdot n^2 - 2 \cdot n^2 = 1/\#$  (which yields the fourth notional equation,  $\# \cdot \# = \#$ , upon inversion), and so forth. In short,  $1/\#$  is isomorphic to the familiar magnitude 0 within the finite part of  $F^\#$ , so  $1/\#$  will now (for clarity) be called 0. Also via  $In$ ,  $\# - \# = F^\#$  and  $\#/\# = F^\#$ , and clearly  $\#/\# = \# \cdot 0 = 0/0$ , so we may now see (to pick up a point from §1) that dividing  $1 \times 0 = 2 \times 0$  by 0 within  $F^\#$  just yields  $F^\# = F^\#$ .

Reiterating those arithmetical operations would be consistent, as a few typical equations (skipped for brevity) would show, so the coherence of  $\#$  is already indicated (to some extent) by the algebraic strengths of  $N^\#$  and  $F^\#$ . The most natural way to extend  $F^\#$  would be by adjoining irrationals (and infinitesimals, see §7), because  $\# \cdot 0$  should include all such numbers: if  $k = \#$ , then lines of arbitrary length are  $\#$  points, each of length 0; and there are similarly geometrical reasons why  $\#$  and 0 should be values of  $\# \cdot 0$ , because a line of  $\#$  points is 0% of an area, which has  $\# \cdot \# = \#$  points. You will see (in §5) that such extensions could retain the algebraic strengths of  $F^\#$ , but for brevity the negative numbers will be adjoined next, because a resulting algebraic structure, the number pitch (defined in §5), extends the other number fields just like it extends the rationals (revealing more of the coherence of  $\#$ ).

#### 4. Dividing by Zero Vectors.



It is useful to assign numerical coordinates to a line's points, relative to two arbitrary points labelled 0 and 1 (see §7), and so it is quite natural (in the current context) to regard the adjunction of negatives as the introduction of two directions,  $\pm 1$ . So consider two signed collections  $+F^\#$  and  $-F^\#$  defined by  $+x @ +F^\# \text{ iff } x @ F^\# \text{ iff } -x @ -F^\#$ , with the familiar properties of signs, e.g.  $-x = -y \text{ iff } x = y$  and  $(-x) \cdot (-y) = +(x \cdot y)$ , following from the natural properties of directions. The equations for  $+\#$  are those for the fractional  $\#$ , reading  $r$  as  $+r$ , etc., while the equations for  $-\#$  follow from considering the fractional equations in the direction  $-1$  instead of  $+1$  (e.g.  $-\# \pm -x = -\#$ , for  $x @ F^\#$ ). Addition and multiplication may remain commutative and associative (as is easily checked), so the arithmetic of  $+\#$  and  $-\#$  follows from that of  $\#$ . E.g.,  $+\# - +\#$  includes all the rationals and  $+\#$  (via  $In$ ) so,  $+\#$  being an atom of  $+\# - +\# = +\# + -\#$ , therefore  $-\#$  also yields a mere-sum that includes  $+\#$  when added to  $+\#$ , and so  $+\# - +\# = [+F^\#, -F^\#]$ . The remainder of the signed arithmetic is mostly that straightforward, but there is one odd-looking result, because  $(+\#) \cdot (+0) = +F^\#$ , whereas  $(+\#) \cdot (-0) = -F^\#$ , which means that  $+0$  (i.e.  $+(1/\#)$ ) is not quite the same as  $-0$ .

Nonetheless the rational equation  $0 = (-1) \cdot 0$  is obtained by replacing  $\underline{0} =_{\text{df}} [+0, -0]$  with an individual object (not necessarily a pair-set) that relates to the other numbers just like  $\underline{0}$  does and which will be called  $0$  when the positive numbers are called by their previous (unsigned) names. That is not inappropriate because the rational  $0$  is not an undirected quantity, not in the way that the fractionals are undirected, so it really does not make less sense to think of it as having all the directions (of the domain) rather than none. Furthermore, approaching the rational  $0$  via  $\underline{0}$  coheres with other consequences of  $ha_1$  and  $ha_2$ , such as the existence of infinitesimals (see §7), which can have either sign. Of course, if replacing  $\underline{0}$  by a single isomorphic object was a particularly unnatural thing to do, then the plausibility of  $k = \#$  would be challenged, but the main thing is that  $\underline{0}$  is indeed isomorphic to the rational  $0$  (and the mainstream approach is less natural, e.g. its integers are equivalence classes of pair-sets of finite ZF ordinals).

That isomorphism follows from how rationals are not changed by the addition or subtraction of  $\underline{0}$ , and how any rational times  $\underline{0}$  equals  $\underline{0}$ , as follows. From  $+F^\#$ ,  $+0 + +0 = +0$ , so  $+0$  is an atom of  $+0 - +0 = +0 + -0$ , and so  $-0$  is too, and nothing else is, so  $+0 + -0 = \underline{0}$ , and furthermore (from  $-F^\#$ )  $-0 + -0 = -0$ , so  $\underline{0} + \underline{0} = \underline{0}$ , and so  $\underline{0} \pm \underline{0} = \underline{0}$  because  $-\underline{0} = \underline{0}$ . More briefly now, (from  $+F^\#$ )  $+r \pm +0 = +r$ , and (from  $-F^\#$ )  $-r \pm -0 = -r$ , and similarly with  $n$  instead of  $r$ , and furthermore  $(+r) \cdot (+0) = +0$  and  $(+r) \cdot (-0)$

$= -0$ , so  $(-r) \cdot (-0) = +0$  and  $(-r) \cdot (+0) = -0$ , again with  $n$  instead of  $r$ . Finally, directly from the properties of signs,  $\underline{0} \cdot \underline{0} = \underline{0}$ .

So, with  $\underline{0}$  replaced by an isomorphic atom called  $0$ , and the positive numbers called by their old names, the new domain consists of  $\#, -\#$  and all of the rationals. Division by  $\#$  and  $-\#$  are still multiplication by  $+0$  and (respectively)  $-0$ , by definition, so multiplication by  $0$  is now division by  $[\#, -\#]$ , and  $1/0 = [\#, -\#]$ . Although it was the case within  $F^\#$  that  $1/0 = \#$ , such differences between domains are not too unusual, even within school mathematics, cf. how square numbers (e.g. 1, 4, 9) each have one square root in  $N$ , but two in  $Z$  (where  $x @ Z$  iff  $x \in \mathbf{Z}$ , the informal set of integers that we learnt about at school, and which is arithmetically isomorphic to the  $ZF$  set of integers). What is more of a problem is that although  $\# - \#$  now equals the whole domain,  $\#/\#$  is only the non-negative part of it. A more useful structure therefore results from replacing both  $\underline{0}$  and  $\Theta =_{df} [\#, -\#]$  by new atoms.

That structure is the rational number pitch, in which  $1/0 = \Theta_P$  (defined in §5). But before I define that algebraic structure, note that although  $Z$  was bypassed as the number systems were built up via  $F^\#$ , that was not because of any inconsistency between  $Z$  and  $\#$ . In fact, because  $1/0 + 1/0 = \Theta + \Theta = \# - \# = 0/0$  (which will become  $\Theta_P + \Theta_P = 0 \cdot \Theta_P$  below), the familiar rules for adding and multiplying ratios of integers, i.e.  $(w/x) + (y/z) = (w \cdot z + x \cdot y)/(x \cdot z)$  and  $(w/x) \cdot (y/z) = (w \cdot y)/(x \cdot z)$ , may now remain valid when  $w, x, y$  and  $z$  are *any* integers, and of course, being able to round out the validity of familiar rules indicates coherence. Furthermore, that particular example occurred because multiplication by  $\pm\#$  (below,  $\Theta_P$ ) cannot distribute over addition now that the subtraction of a number is the addition of its negative. So such extensions of validity compensate somewhat for (and thereby indicate the coherence of) that algebraic weakness.

Coherence is similarly indicated by situations that involve to consider exponentiation in any breadth it is apposite to note that  $0^{(2-1)} = 0$  and  $0^{(1-2)} = \Theta$  (below,  $\Theta_P$ ), whereas  $0^2/0 = 0/0 = 0/0^2$ , so that the extension of the familiar rule  $z^{(x+y)} = z^x \cdot z^y$  to include  $z = 0$  is the weaker rule  $z^{(x+y)} \underline{\leq} z^x \cdot z^y$  (cf. mere-distributivity in §5). But that weakness allows  $0^0$  to equal 1 instead of  $0/0$ , and it can be useful to stipulate that  $0^0 = 1$ , e.g. when algebraically manipulating polynomials (cf. Kaplan 1999, p. 169) or when recursively defining exponentiation. Furthermore, a relatively natural (if quasi-multifunctional) way to handle rational powers is via biconditionals such as  $x @ y^{1/2}$  iff  $x^2 = y$ . Then  $y^{(1/2 + 1/2)} = y \underline{\leq} [y, -y] = y^{1/2} \cdot y^{1/2}$ , and the rule  $(y^{1/2})^2 = (y^2)^{1/2}$  can be kept even when  $y$  is negative; whereas the familiar root

function, say  $\sqrt{\phantom{x}}$ , takes only positive values, so  $(\sqrt{x}) \cdot (\sqrt{y}) = \sqrt{(x \cdot y)}$  must fail when  $x$  and  $y$  can be negative (e.g. becoming  $-1 \neq 1$  when  $x = y = -1$ ). Note that although  $\sqrt{((1)^2)} = 1$  is certainly better looking than  $((1)^2)^{\frac{1}{2}} = [1, -1]$ , less attractive is  $\sqrt{((-1)^2)} = 1$ .

### 5. One Pitch, Two Teams.

Some algebraic structures have naturally appeared, so in this section I shall define the *pitch* and *team* structures (to refer to in §7). The substructure of the arithmetic of  $[F^\#, -F^\#]$  within which  $\#$  and  $-\#$  only occur in the forms  $\underline{0}$  and  $\Theta$  will be called the *rational number pitch* because (i) it contains the rational number field and (ii) any field may be extended to its corresponding pitch, as follows. A number field  $\mathbf{F}$  is usually a set  $\mathbf{F}$  of numbers together with two arithmetical operations that satisfy the familiar field axioms. But an isomorphic structure is therefore possessed by a mere-sum  $\Phi$  given by  $x @ \Phi$  iff  $x \in \mathbf{F}$ , when  $@$  replaces  $\in$  in those axioms. Adjoining a number  $\Theta_\Phi$  (with the following properties) to the field  $\Phi$  makes the number pitch  $\Phi^\ominus =_{\text{df}} [\Theta_\Phi, \Phi]$ . The arithmetical operations are extended by the following 6 equations (where  $x @ \Phi$  and  $x \neq 0$ ).

$$\begin{array}{lll} \Theta_\Phi + 0 = \Theta_\Phi & \Theta_\Phi + x = \Theta_\Phi & \Theta_\Phi + \Theta_\Phi = \Phi^\ominus \\ \Theta_\Phi \cdot 0 = \Phi^\ominus & \Theta_\Phi \cdot x = \Theta_\Phi & \Theta_\Phi \cdot \Theta_\Phi = \Theta_\Phi \end{array}$$

Also, division by 0 is multiplication by  $\Theta_\Phi$ , and *vice versa*, and the subtraction of  $\Theta_\Phi$  is the same as its addition, and addition and multiplication both remain commutative and associative within the pitch, which is therefore rather neat. Pitches are mere-closed under addition, subtraction, multiplication and division. Consistency is easily shown by a few equations such as, for an example of associative multiplication,  $0 \cdot (0 \cdot \Theta_\Phi) = 0 \cdot \Phi^\ominus = [0 \cdot \Theta_\Phi, 0 \cdot \Phi] = [\Phi^\ominus, 0] = \Phi^\ominus = 0 \cdot \Theta_\Phi = (0 \times 0) \cdot \Theta_\Phi$ . And the only algebraic cost of extending a field to a pitch is what I call *mere-distributivity*, i.e. if  $x, y$  and  $z$  are atoms of  $\Phi^\ominus$ , then  $x \cdot (y + z) \leq x \cdot y + x \cdot z$ , with equality (distributivity) only if  $x \neq \Theta_\Phi$ .

In particular, when  $\mathbf{F} = \mathbf{Q}$  (the rational number field), adjoining  $\Theta_P$  to  $\Phi = \mathbf{P}$  (*rho*, for rational, or Pythagoras) yields the pitch  $\mathbf{P}^\ominus$ , which is the same structure that replacing  $\Theta$  by an isomorphic atom (in §4) would yield, as is easily checked. The previous paragraph therefore provides a summary of the previous sections, whilst being applicable to the other number fields as well. Let the field  $\Delta$  (*delta*, for Dedekind) be defined by  $x @ \Delta$  iff  $x \in \mathbf{R}$  (the real number field). The adjunction of  $\Theta_\Delta$  to  $\Delta$  yields the pitch  $\Delta^\ominus$ .

However, lines of  $\#$  points occur in two possible cases, C-I and C-II (see §6), corresponding to  $N$  being an actual (completed, finitesque, combinatorial) infinity or (respectively) a potential infinity, and in C-II only reals that could (in principle) be defined by finite laws are legitimate. Denoting such a field by  $\Lambda$  (*lambda*, for legal), the adjunction of  $\Theta_\Lambda$  yields what I will call a *legal* real number pitch,  $\Lambda^\ominus$ . Similarly, let the field  $\Gamma$  (*gamma*, for Gauss) be defined by  $x @ \Gamma$  iff  $x \in \mathbf{C}$  (the complex number field), which is a Gaussian plane. The adjunction of  $\Theta_\Gamma$  yields the pitch  $\Gamma^\ominus$ , which is a projection of a Riemann sphere. In C-II, adjoining the imaginary unit  $i$  to  $\Lambda$  yields a *legal* complex number field,  $I$  (*iota*, for imaginary), with  $i$ 's adjunction to  $\Lambda^\ominus$  yielding  $I^\ominus$ . Incidentally, had irrational magnitudes been adjoined to  $F^\#$  (in §3) both  $\Theta_\Delta$  and  $\Theta_\Lambda$  would also have replaced  $[\#, -\#]$ , just as  $\Theta_P$  did, while  $\Theta_\Gamma$  and  $\Theta_I$  would have replaced all the  $\# \cdot e^{i\theta}$  for  $0 \leq \theta < 2\pi$  (legal  $\theta$ , in the case of  $\Theta_I$ ).

A precise description of the increase in *symmetry* caused by the adjunction of  $\Theta_\Phi$  to a field is facilitated by defining the following structure,  $\langle T, e, \mathbf{a}, \mathbf{M} \rangle$ , which I call the number *team*  $T$ . Teams are so-called because they are commutative generalizations of Abelian (i.e. commutative) *groups*, e.g.  $\langle \mathbf{Z}, 0, +, \emptyset \rangle$  is an improper team, as follows.  $T$  is any mere-sum of numbers that is mere-closed under an associative and commutative arithmetical operation  $\mathbf{a}$ , with an identity  $e @ T$  such that, for each  $x @ T$ ,  $e \mathbf{a} x = x$  and there is a  $y @ T$  such that  $e @ x \mathbf{a} y$ . The finite set  $\mathbf{M}$  contains those  $x$  for which that last  $@$  *cannot* be replaced by equality, teams being ‘proper’ if  $\emptyset$  is a proper subset of  $\mathbf{M}$ , e.g. the proper multiplicative team of the fractionals is  $\langle F^\#, 1, \times, \{0, \#\} \rangle$ . So, whereas a field  $\Phi$  contains an additive Abelian group  $\langle \Phi, 0, +, \emptyset \rangle$  and a multiplicative commutative monoid, a pitch  $\Phi^\ominus$  is relatively symmetrical because it contains two proper teams,  $\langle \Phi^\ominus, 0, +, \{\Theta_\Phi\} \rangle$  and  $\langle \Phi^\ominus, 1, \times, \{0, \Theta_\Phi\} \rangle$ .

## 6. Another Continuum Problem.

After all that algebra, perhaps a brief recap would be useful. The arithmetic of  $\#$  was deduced (in §3) from two assumptions, ( $ha_1$ ) that  $\#$  is the number of points in a line, and ( $ha_2$ ) that  $1/\#$  is the length of a point. (The symbol  $\#$  was chosen because it illustrates one intuition for the existence of points within lines.) Directions were then given to the fractional magnitudes (in §4), whence the isomorphism between  $[(1/\#), -(1/\#)]$  and the rational 0 led to the replacement of  $[(\#/1), -(\#/1)]$  by a new number  $\Theta_P$ . (The sym-

bol  $\Theta$  was chosen because an ideal point at infinity turns an infinite line into an infinite circle, cf. §7.) There were several signs of the coherence of  $\#$  with our elementary number systems. But, as mentioned in §5, it will make a difference whether the infinitude of  $N$  is actual or potential, when coordinates are given to a line of  $\#$  points (in §7), so in this section I will look at those two possibilities. As mentioned in §2,  $\#$  must be a non-set-theoretic cardinal. Although  $0 \cdot \# \neq 0$ , whereas  $0 \cdot \aleph_\alpha = 0$  for every ordinal  $\alpha$ , there are still two possibilities, as follows. Either (C-I)  $\#$  is *bigger* than every  $\aleph_\alpha$ , or else (C-II)  $\#$  is cardinally *incomparable* with every  $\aleph_\alpha$ . In C-I, lines of  $\#$  points would contain all transfinite cardinalities of points, which is a lot of points (but then, 0 is *very* small); and in C-II, lines of  $\#$  points could not even contain  $\aleph_0$  points, which is to say that the infinitude of  $N$  is *potential* (a concept that is usually associated with constructivism, but which has also been associated with proper classes, see Hart 1976).

I call the choice between C-I and C-II *another* continuum problem because of Cantor's famous continuum problem, which concerns the unresolved details of standard cardinal exponentiation (see Feferman *et al.* 2000). So, before looking a little closer at C-I and C-II (although a detailed comparison must await such developments as those mentioned in §7) I will glance at cardinal exponentiation involving  $\#$ . A simple 'diagonal argument' involving the diagonal of a geometrical square shows that the number of ways in which a 0 or a 1 may be associated arbitrarily with each point of a line of  $\#$  points is a number bigger than  $\#$ , say  $\#^+$ , not  $2^\#$  because that notation has already been used for the kind of exponentiation that extends the field operation (e.g.  $0^0$  in §4) and there is no obvious isomorphism. Now, although  $\#^+$  shows that  $\#$  is increasable (cf. the transfinites), so that within an appropriately extended number system  $\#^+$  would be one of the values of  $1/0$  (cf.  $-\#$  becoming part of  $1/0$  in §4), with  $\#^+ + \#^+ = \#^+$  and  $\#^+ \cdot \# = \#^+$  etc. (cf. transfinite cardinal arithmetic), nonetheless since  $\#^+$  does not directly concern continuity such arithmetic is not pursued here.

The first thing to note about C-I is that, within any reasonable theory of (well-ordered) classes, the Cartesian product of the null-class with any class is likely to be the null-class, so  $\# = \Omega$  also seems unlikely. (Given the axiom of choice, we can rule out  $\# = 2^{\aleph_0}$ , of course.) One heuristic principle of Cantorian set theory is that "any potential infinity presupposes a corresponding actual infinity" (principle *a* of Hallett 1984, p. 7), which might imply, intuitively, that  $\# > \Omega$ . But another Cantorian principle is that  $\Omega$  "cannot be mathematically determined" (principle *c* of Hallett 1984, p. 7). Whilst being actual (in the sense of principle *a*),  $\Omega$  cannot be as fi-

nitesque (in the sense of principle *b*) as the transfinites, which might imply that lines of  $\#$  points would *not* also contain  $\Omega$  points. It might therefore be appropriate to call  $\Omega$ 's infinitude *potential*, although Cantor called it *absolute*. In short, proper classes are quite mysterious, and so little can be said pre-theoretically about (e.g. against) C-I at present.

Looking ahead to C-II, note that  $\Omega$  is often likened to a potential infinity because of ZF's hierarchical nature. ZF's axiom of infinity is just the smallest of several axioms of infinity (see Feferman *et al.* 2000) and without it the natural numbers would form a proper class, just as the totality of all the ZF sets would be a set within a set theory containing a large cardinal axiom. (Assuming that each transfinite is a proper number, one might wish to consider an arbitrary subcollection of Cantorian cardinals, and hence the totality of all such subcollections, which would be cardinally larger than  $\Omega$ .) In C-II,  $N$  would effectively be a potentially infinite kind of totality, at least by comparison with a line of points. It is certainly the repeated addition of 1 that yields the names of the finite cardinals in  $N$ , starting from 1, via  $2 =_{\text{df}} 1 + 1$ , and  $3 =_{\text{df}} 2 + 1$ , and so forth, so their totality (i.e. all of *those* cardinals, the ones with such names) is defined in an endlessly hierarchical kind of way, which is quite different to the way that lines are full of points (cf. §2). So it is logically possible that, although there are  $n$  points, for any (natural number)  $n$ , in a line of points, there are not  $\aleph_0$  points, just because of the endlessly hierarchical way in which  $N$  is defined (pre-theoretically).

C-II is a counter-intuitive possibility, but it has therefore been overlooked and has not, in particular, been refuted. In fact, good reasons why  $N$  should act finitesquely (as  $\mathbf{N}$  does) that are not also reasons why the proper class of all the cardinals should *are* rather elusive (cf. Fletcher forthcoming). Even when  $N$ 's infinitude is regarded as actual, the concepts of cardinal and ordinal begin to diverge with  $\aleph_0$  and  $\omega$  because of  $\mathbf{N}$ 's endlessness, so that endlessness is certainly able to cause *some* shift away from finitesque behaviour. And the approach of the natural numbers to  $\aleph_0$  does resemble the approach of the cardinal numbers to  $\Omega$ , even though  $\Omega$  cannot be as actually (or finitesquely) infinite as  $\aleph_0$ . So note that the possibility of C-II just requires that two infinite collections (the endless sequence of *the* natural numbers and the *primitive* line of points) that are even more different in kind (than  $\aleph_0$  and  $\Omega$ ) *might* differ significantly. Furthermore, although  $\#$  is not necessarily a number at all, it would, were it a number, be primarily a possible number of points, and so arguments that  $\aleph_0$  is better at being a number of *numbers* than a number of spatio-temporal *objects* (e.g.

Cooke 2003) amount to an argument for C-II.

### 7. Infinitesimals.

Admittedly, neither C-I nor C-II appears particularly attractive, intuitively, but that may just be the way with lines of points (cf. the famous Banach-Tarski paradox, and also Freiling 1986). So, neither case having been considered in any detail yet, let alone refuted, the points of an infinite line (satisfying Hilbert's axioms of incidence, order and congruence) will now be given numerical labels, under the assumption of  $k = \#$ . Calling an arbitrary point 0 gives us our *origin*, and calling any other point 1 defines a *unit* of length and a *positive* direction. Any point, say  $p$ , between the points 0 and 1 lies in one of the tenths of that interval (e.g. between the points called 0·0 and 0·1), and in one of the tenths of that tenth (e.g. between 0·00 and 0·01), and so forth, and is therefore associated with an endless decimal expansion, say  $d(p)$  (e.g. 0·000...), which is basically a real number.

In C-I,  $d$  is a structure-preserving function that maps each point that is a finite distance from 0 to a real number. If just one point mapped to each real, the number of points would be  $2^{\aleph_0}$ , and so if  $\# \neq 2^{\aleph_0}$  (e.g. via the axiom of choice) then there are infinitesimals in C-I. Any point, say  $i$ , apart from 0 but with  $d(i) = 0$ , could be used to define an infinitesimal unit of length. The uniformity of the line means that  $i$  might have been chosen as the point called 1 (above, following our choice of origin), and so the point that was actually called 1 shows (under the alternative labelling) that there are also points infinitely distant from 0. Such points can be given the numerical label  $\Theta_\Delta$  (see below). Lines are therefore partitioned by the function  $d$  from any point  $q$  to some  $d(q) @ \Delta^\ominus$ , in C-I, and so the real number line is quite a good mathematical model of the primitive line in that case. (Number lines corresponding more precisely to C-I will not be considered, because this essay is primarily concerned with introducing the possibility of  $k = \#$ .)

In C-II, the endless decimal expansions would be potentially infinite, so the arbitrary expansions that would be associated with *most* points would be impossible to identify, even in principle, except *via* those points. Consequently, the most analytically useful reals would be those whose expansions could be specified individually by finite laws, which I call the legal ones (e.g., recursive or computable ones; cf. Weyl's line, although it was not composed of points, e.g. see Bell 2000). Legal functions would not include the classical monsters, but could include Dirac's useful delta func-

tion because there are also infinitesimals in C-II, as follows. In any infinitely extended line of points, there are points that are  $n$  unit lengths from 0 for *every*  $n$ , but in C-II there is no sequence of *all* such lengths (since there are not  $\aleph_0$  points) and so there are points that are infinitely distant from 0 (relative to any unit of length). Such points are naturally associated with  $\Theta_\Delta$  (see below), and because one of those points might have been called 1 originally, there are also infinitesimals (via the converse of the argument above).

Since the coherence of  $\#$  is indicated by the use of  $\Theta_\Delta$  or  $\Theta_\Lambda$  to label points infinitely distant from 0, I shall briefly justify that use of  $\Theta_\Delta$  (the case of  $\Theta_\Lambda$  being analogous). For real  $x \neq 0$ , all of  $\Theta_\Delta + 0$ ,  $\Theta_\Delta + x$ ,  $\Theta_\Delta \cdot x$  and  $\Theta_\Delta \cdot \Theta_\Delta$  must equal  $\Theta_\Delta$  (see §5), where the additions correspond to vector additions (e.g. the first corresponds to going an infinitesimal distance from a point infinitely distant from 0, which amounts to remaining infinitely distant from 0) and similarly for the multiplications. So the other three equations, and also the commutative and associative laws, are clearly satisfied. Also required is  $\Theta_\Delta + \Theta_\Delta = \Delta^\ominus$ , so consider any two points infinitely distant from 0, on either side of 0; going an infinite distance from one point back towards 0 (and possibly beyond it) could amount to being at any other point. And  $\Theta_\Delta \cdot 0 = \Delta^\ominus$  is clearly satisfied too because the multiplication of an infinitesimal magnitude with an infinite magnitude may result in any magnitude, of positive (or negative) sign if the signs of the multipliers are the same (respectively different).

Incidentally,  $\#$  and  $-\#$  might be used to represent the two infinite regions separately, but using  $\#$  to label points at infinite distances from 0 is *not* to say that such points are  $\#$  units from 0, no more than using 0 to label  $i$  amounts to saying that  $i$  is *not* distinct from 0. Furthermore, coherence may also be indicated by the use of  $\Theta_\Delta$  (or  $\Theta_\Lambda$ ) and  $\Theta_\Gamma$  (respectively  $\Theta_I$ ) within other mathematical structures that, assuming C-I (respectively C-II), resemble division by 0. The ideal point  $\infty$  at infinity in extended lines, for example, could be called  $\Theta_\Delta$  (respectively  $\Theta_\Lambda$ ), with the planar  $\infty$  becoming  $\Theta_\Gamma$  (respectively  $\Theta_I$ ). And  $0/0$  is already used to denote indeterminate forms in the calculus. In C-II, lines contain  $n$  points for any  $n$ , and  $\#$  points, but no intermediate amounts, so  $\Theta_I$  would be a natural choice for unbounded complex limits, in that case, with  $\pm\#$  denoting unbounded real limits, such as the gradients of vertical lines.

Of course, infinitesimals are certainly counter-intuitive, but it is not, given the *extreme* smallness of 0, *especially* counter-intuitive that infini-



tesimals should occur in lines of points. One argument against infinitesimals (cf. Grattan-Guinness 2000, p. 236, and Moore 2002, p. 325) concerns a point-object going  $I =_{\text{df}} (i + i + i + \dots)$  from 0, and therefore going just as far as one going  $I$  from  $n \cdot i$ , because  $n \cdot i + I = I$ . Nonetheless, in C-II such counter-intuitive situations cannot arise, while in C-I we may simply deduce that  $I$  is undefined. It would be appropriate to regard  $I$  as undefined because  $(1 + 1 + 1 + \dots)$  is similarly undefined, and furthermore the Banach-Tarski paradox (which follows from the Banach-Tarski theorem of real analysis via C-D) is usually resolved by deducing that the counter-intuitively decomposed sphere's parts are measureless. Note that the *absence* of infinitesimals (and their reciprocals, see below) is therefore associated with a *similar* counter-intuitiveness.

Consider a rocket going a metre in 1 second, another metre in  $\frac{1}{2}$  second, another in  $\frac{1}{4}$  second, etc., along a straight line within an infinite 'flat' space containing no infinitesimals (e.g.  $\mathbf{R}^3$ ). It appears that the rocket should vanish, or at least teleport, after 2 seconds (see Saari and Xia 1995, for similar vanishings from a Newtonian space). But intuitively, the rocket would neither vanish nor teleport, and we might prefer to imagine it reaching instead an infinitely distant part of space (vanishing only from a Euclidean universe of discourse). Were  $k = \#$ , an infinite 'flat' space would, by containing such infinitely separated places, be more like the space of projective geometry, which is the most symmetrical of the geometries (in their group-theoretic classification). So note that symmetrical structures do seem more likely than asymmetrical ones to be physically instantiated (cf. Penrose 2000, pp. 230-231; see also Castellani 2002).

Infinitesimals are often regarded as unrealistic, but the basic concept cannot be too incoherent because several formal kinds, e.g. 'hyperreal', 'surreal' and 'smooth' ones, already exist (primarily to assist standard analysis). Those in lines of  $\#$  points, say 'irreal' ones, are not being posited for their utility, but if lines might contain  $\#$  points then it would obviously be useful to know more about them. So inevitably the issue of axiomatization arises. A good set of axioms for  $\#$  would require not only more of  $\#$ 's informal properties than I have been able to mention in this essay, but also a good formal language. That would presumably be a mereological one (judging by the above) but such languages are still in the process of development (see Forrest 2002) and while a mereological approach would probably facilitate the comparison of C-I, C-D, C-II and the cases of lines *not* being full of points, which would clearly be useful, it is precisely because such comparisons would be useful that the choice of a formal lan-

guage ought to depend upon what the *other* options for  $k$  are. In short, even more informal metaphysics (such as the above) should precede the formal mathematics of  $\#$ .

To sum up, there could be physical continua, for all we really know, and lines may well be full of points. One coherent (if occasionally counter-intuitive) metaphysical possibility is of course  $k = 2^{\aleph_0}$ , but another may well be  $k = \#$ . Since lines of  $k$  points may have any length, it is hardly counter-intuitive that  $0 \cdot k$  should equal  $0/0$ . And continuity is a relatively simple notion, so the informal coherence of the elegant hypothesis  $k = \#$  (demonstrated above) makes it relatively plausible that the essence of continuity is captured by  $0 \cdot k \neq 0$ . Whether or not points exist, there are therefore strong indications that we would be wise to take a mereological approach to the metaphysics of spaces and classes. Furthermore, the introduction of the concept of  $\#$  may also contribute, indirectly, to the plausibility of the natural numbers forming a (non-constructive kind of) potential infinitude, which is a concept usually associated with lines *not* being full of points.

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# REVIEWS

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## **Resurrecting, Re-examining and Reburying the Great Fact**

A Critical Review of Stephen Neale, *Facing Facts*,

Oxford: Clarendon Press 2001, xv, 254 pp., bibl., index, glossary.

by Herbert Hochberg

After decades of being relegated to the philosophical wastelands as useless verbal byproducts, facts, like properties, have again become respectable entities. The hygienic disposal of facts by the analytic techniques of philosophers of language was evident in D. Davidson's banishing facts with a flick of the phrase: "...if true sentences correspond to anything, they all correspond to the same thing." (2001, 184) This, he concluded, trivialized the notion of *correspondence*, long associated with the reliance on facts as basic grounds of truth. Noting P. Strawson's well known rejoinder to J. Austin, he approvingly cited Strawson's having correctly "gone on to claim that 'while we certainly say that a statement corresponds to (fits, is borne out by, agrees with) the facts' this is merely 'a variant on saying it is true'." (2001, 184) One can almost hear the Oxonian intonation echo from the phrase "we certainly say," which supposedly sufficed to reduce "it corresponds to the facts" to "it is true." That familiar philosophical style of the 1950s was seemingly buttressed by a devotion to Convention-T and "formal semantics." The resulting union of dismissive ordinary language analysis with cryptic Tarskian formal semantics sufficed to bar the turn to serious metaphysical issues concerning *facts* and *truth-makers* to many minds for years. Echoes of that barrier, like background noise, are still with us in the form of so-called "minimalist theories" of truth.

Strawson's dismissive formula reversed the familiar parsing of "is true" in terms of "corresponds to a fact" that characterizes at least one variation of a correspondence theory of truth. His formula is still frequently put to such use, as the turning of the phrase supposedly exhibits the vacuous circularity of correspondence theories—that circularity being established by what "we certainly say" and, of course, by what "we would not say." As Convention-T was thought to suffice for what was needed regarding "truth," it became sufficient to note that predicates were "true of" objects, and thus represented nothing, neither properties nor relations, beyond the objects they were "true of." The world was a world of objects and phi-

losophers were suitably focused on the relations between “word and object.” This was supposedly so as long as one’s logic was confined to first order logic. In such a logico-linguistic setting the classical problems posed by “predicates” and predication did not even arise, since it was understood that “to be is to be the value of a variable.” What those *values*—objects—were taken to be became a matter of selecting a conceptual scheme for accommodating the statements of science, “to the extent that (a) philosophy of science is philosophy enough and (b) the ... logical underpinnings of science do not engender new philosophical problems of their own.” (Quine, 1953, 46) Guided by what “we certainly say,” on one side of the Atlantic, and the devotion to “semantic ascent,” on the other, many turned their backs on the classic ontological issues concerning the link between truth-bearers and truth-makers, the analysis of facts, and the problems posed by *intentionality*—issues that had been a focus of British and Austrian philosophy in the early decades of the 20<sup>th</sup> century and gave rise to the “analytic tradition” fostered in England by Russell and Moore.

### *I. The Return of The Great Fact*

S. Neale proposes to consider and face facts—by which he means pointing out the need for those who speak about facts to face certain problems requiring basic semantic revisions that will permit them to safely do so. As he sees supposedly formidable arguments of a semantic nature to lie in wait for “the friends of the facts,” he sets out to explain the nature of the problems facing those who would face facts. Thus the book embarks on a lengthy story focused on a familiar offspring of Convention-T and truth-functional logic—The Great Fact—which can be seen as a contemporary mutation of a line of argument, spawned at the close of the 19<sup>th</sup> century by the Absolute Idealism of F. H. Bradley and B. Bosanquet, and found in Frege’s writing. Arising in different forms, and sometimes more ominously dubbed “The Slingshot,” it has frequently been disposed of, only to reappear and continuously play a prominent role in attacks on correspondence and representational theories of truth that are linked to modern realism with respect to facts.

*Facing Facts* repeats an earlier 1995 paper that appeared in *Mind*. Neale mentions the sixty-five page paper in presenting a history of the development of his book in a two page preface in which over a half of a page lists names of prominent philosophers and logicians who “bombarded” him “with questions and advice.” The reader learns that the book was needed to

set out “a more detailed exposition of my 1995 proof,” since the published paper was a “pruned version” required by the need to “produce a journal-length version.” (Neale, 2001, x) The earlier version, like the book, relies on additional rules about description operators and offers a “proof” of a version of The Great Fact argument that is dependent on new rules for descriptions that are supposedly treated in accordance with Russell’s theory.

Neale’s book purports to do three things. First, to give a detailed and careful consideration of the argument(s) and the problems they put in the way of acknowledging facts. Second, to put *the argument* in a new, resurrected and revitalized form (two actually) by introducing semantic rules for Russellian style definite descriptions, while noting and examining various forms in which the argument fails, and why. This will purportedly enable Neale to show that “it” is obviously valid, since he will *demonstrate* that the additional rules are justified by arguments that show them to amount to *derived rules* in a Russellian-style system. For, supposedly, the new rules will be shown to be based on employing nothing more than an innocent notational shorthand. Thus, some notational shorthand, *and only that*, is purportedly codified in a framework that basically employs Russell’s analysis of definite descriptive phrases in context, as in *Principia*, and a standard predicate logic. Doing this, Neale claims to prove, third, that The Great Fact, in his resurrection of it, poses an ominous threat to correspondence theories of truth and facts. Threatened by The Great Fact, those who would face facts must meet the threat by developing an adequate semantics for speaking of and representing facts. One cannot ignore The Great Fact as a harmless nuisance that merely embodies the point that materially equivalent statements are allowed to replace each other *salva veritate* in standard “extensional” contexts.

In Neale’s scenario, the resurrected Great Fact, phrased in a currently fashionable linguistic manner, arises to haunt the friends of the facts—like a spectre from a third-rate film that returns decade after decade. The proper rites for disposing of it once and for all have yet to be devised and applied. Those rites, on Neale’s account, will involve an adequate semantics for descriptions that will, in turn, allow undisturbed discourse about facts. Neale’s book takes steps towards accomplishing the first task—hence his focus on, and fanfare about, his introduction of rules governing substitution, instantiation, etc. for definite descriptive phrases. It would thus appear that the book is written by one with the courage to face facts and the threat to them—one who, in a sympathetic way, seeks to impress on those who ac-

cept facts the need for potent semantical amulets to ward off the return of The Great Fact.

Yet, despite the friendly face turned to facts, and the running criticisms of versions of The Great Fact argument, by R. Rorty, Davidson, and others, the heart of the book, after some 170 pages of elaboration, containing lengthy and cumbersome, if not complicated, formulae, is the reintroduction onto the philosophical scene of a supposedly properly formulated version of The Great Fact. Even given the friendly motive of scaring friends of the facts into doing the work that Neale proclaims they need to do to develop their “semantics,” the basic claim that emerges is that The Great Fact argument, as resurrected by Neale, *is valid*. Thus we will be dealing with, and focus on, that high point of the book—his “proof.” That purported proof, we will see, fails to give new life to the tired ghost of an argument. Neale no more offers a “proof,” by his lengthy and elaborate presentation, than Davidson did by repeatedly reiterating variations of a transparently invalid version of it. Yet, his argument is stylistically different, and in an interesting and informative way. For his line of argument helps one to see just how the misstep is hidden, apparently even from one who takes that step, like Neale himself. Thus, for those interested in facts and the current disputes about theories of truth and truth-making, the chief interest of Neale’s book is not to be found in the reappearance of the problem that supposedly besets attempts to take propositions or beliefs to have their truth value depend on “pieces” of the world, and not simply on The World as a whole. Rather, it lies in the “meta-argument” he offers to justify the rules that he introduces to govern the use of definite descriptions. That argument makes it clear just how one can be unwittingly led down the circuitous path of thought trodden by believers in The Great Fact.

To begin, consider a simple variant of The Great Fact argument. With “S” and “T” being any true sentences, we take as premises:

- (I) 1. S  
 2. T  
 3. “S” designates the-fact-that-S—alternatively, “S” is made true by the fact S, “S” represents the-fact-that-S, etc. [In what follows below “S” is used without quotes to represent the fact.]

We then supposedly can derive “‘S’ designates (is made true by, represents, etc.) the-fact-that-T.” This supposedly shows that any true sentence designates the same fact as any other true sentence. Hence, one can conclude that



if there are any facts there is at most one. As C. I. Lewis apparently suggested (and, in his way, F. H. Bradley concluded) the truth-maker of all true propositions (true beliefs) is The World. Hence, *atomic facts*, the entities that became crucial to the early 20<sup>th</sup> century development of the realistic pluralism of Russell, Moore and Wittgenstein—the philosophy of logical atomism, as Russell called it—disappear, absorbed by The Great Fact.

On Davidson's often used presentation of The Great Fact one assumes two "rules" of substitution or replacement. It is assumed, first, that we may replace a sentence by one logically equivalent to it and, second, that we may substitute "coextensive" singular terms. The argument can supposedly be easily adapted to purportedly apply to all candidates for truth-makers, and not just facts. Trivially, one can consider (3) in terms of—"S" is *made true* by the *existence* of the-fact-that-S—or simply consider—"S" designates S. Hence it is not merely an attack on facts but on the very idea of something being a truth ground for a truth-bearer—a correspondent of, or what is represented by, a true statement (proposition) and, hence, that "in virtue of which" the proposition is true. All this assumes that the purported resultant monism is unacceptable.

Like the earlier and well-known versions of the argument Davidson popularized, Neale's variant crucially makes use of abbreviations in context. The Great Fact argument, if simply presented in unabbreviated form, is immediately seen to be trivial and absurd and, hence, not worth serious attention:

- |      |                       |         |
|------|-----------------------|---------|
| (II) | 1*. S                 | premise |
|      | 2*. T                 | premise |
|      | 3*. "S" designates S  | premise |
|      | 4*. "S" designates T. |         |

As presented in (II) the argument is absurd since it obviously uses replacement based on "material equivalence" to derive step (4\*). Moreover, anyone who talks of facts seriously will not take a term that functions like "designates" above in a way that allows for the replacement of materially equivalent contexts—or even of logically equivalent ones. Hence Carnap's attempts to develop a concept of "intentional isomorphism" and Gustav Bergmann's postulating, for a relational predicate transcribed by "means that," that only signs that were different tokens of the same type "meant" the same thing, and thus were substitutable in such "intentional" contexts in an adequate "ideal language." (Bergmann, 1959, 32). Nelson Goodman had set

out an “extensionalist” version of the same claim earlier, based on his extended notion of “extension,” encompassing “primary” and “secondary” extensions of terms: “... this challenge to synonymy was by no means the first but (1) went further than earlier ones by showing that even under an analysis dependent solely on the extensions of terms, every two terms differ in meaning....” (Goodman, 1978, 24; Goodman, 1972) For “two” terms to have the same meaning (be synonymous) they must have the same secondary, as well as primary, extensions, which, Goodman argues, no *two* terms have.

The argument over The Great Fact was never about operating in extensional contexts of standard kinds (substitution of singular terms with the same “referent,” of predicate terms with the same “extension,” of sentence patterns that are materially equivalent). That is why many ignored it or dismissed it as one dismisses a superstitious belief. What it has always been about is whether devotees of The Great Fact could formulate the argument without either overtly appealing to or tacitly employing a replacement that was based on two sentential expressions *in fact* having the same truth value (or, in fact, being true). And that is not at all the same as simply assuming that two arbitrary sentences are true or have the same truth value—premises (1) and (2) above. Even Davidson seemed to take that for granted. Strategically, then, the defender of The Great Fact must avoid making such an appeal, explicitly or implicitly. A traditional, by now almost ritual, detour that has been employed makes use of Russellian definite descriptions, and, in place of (4) in (II), continues from (1)-(3) as follows:

4.  $(\iota x)(x = a \ \& \ S) = (\iota x)(x = a)$
5.  $(\iota x)(x = a \ \& \ T) = (\iota x)(x = a)$
6.  $(\iota x)(x = a \ \& \ S) = (\iota x)(x = a \ \& \ T)$
7. “S” is logically equivalent to “ $(\iota x)(x = a \ \& \ S) = (\iota x)(x = a)$ ”
8. “S” designates  $(\iota x)(x = a \ \& \ S) = (\iota x)(x = a)$
9. “S” designates  $(\iota x)(x = a \ \& \ T) = (\iota x)(x = a)$
10. “T” is logically equivalent to “ $(\iota x)(x = a \ \& \ T) = (\iota x)(x = a)$ ”
11. “S” designates T.

In this detour around the objection to the appeal to material equivalence, the devotees of The Great Fact assume that “a” is a semantically “proper” name designating an object and hence that “ $a = (\iota x)(x = a)$ ” is true, under Russell’s treatment of the iota operator, and even “logically” true or L--true—since it is a consequence of a “semantical” or designation rule in Carnap’s

familiar sense. Each step then supposedly follows from applying substitution rules for “coextensive” singular terms, replacement of logically equivalent sentences or standard logical rules. As Neale, in his way, observes, if the above descriptive phrases are treated, like proper names, as basic patterns in an extensional system, where one takes expressions like “ $x = a \ \& \ S$ ” as predicates with extensions, then the argument is not worth discussing, as the rules governing such terms come into question. In this connection he has an extensive and useful discussion of the point and related matters connected to early comments by Gödel and others.

If descriptions are treated along the lines of Russell’s theory of definite descriptions, then the descriptive phrases are contextually defined signs. Therefore, we should expand them to examine the argument. If we attempt to do so an immediate and obvious question arises about the scope of descriptions in contexts like (8) and (9). Neale discusses such matters as well. Putting that aside, we can take the scope(s) as “secondary,” just governing the sentential expression after the occurrence of “designates.” If one expands the definite descriptions in (4)-(8) it becomes immediately obvious that there is no way to get to (9) unless one is allowed to replace an occurrence of “S” by an occurrence of “T.” In fact the rule about the substitution of co-extensive singular terms becomes irrelevant, since the definite descriptions have “disappeared.” Gödel, as Neale notes, had pointed out quite early that one cannot use the type of argument Davidson employs (which, as Davidson notes, he derived from A. Church), if one employs a definite descriptive operator as a contextually defined sign in Russell’s manner and not as a primitive sign pattern. Church had noted a similar feature in his use of a lambda operator in place of a definite description operator. These matters also recall Russell’s discussion of the case of the expressions “Scott” and “the author of *Waverley*,” and what “George IV wished to know,” in 1905 in “On Denoting.” Russell pointed out then that if the scope of the description was taken to be secondary in the sentence “George IV wished to know whether Scott was the author of *Waverley*,” one could not validly conclude from that that George IV wished to know whether Scott *was* Scott, given that Scott *was* the author of *Waverley*. Thus such a context was one where scope makes a difference, even when the description is fulfilled. Thus Russell can be seen as taking such an “intentional” context as “intensional.” Such complications are involved in the consideration of primary scope governing a context involving a *semantical* term or phrase like “designates” or “is made true by.” But we need not go into those complications here. What is clear, at the outset, is that to replace an occurrence of “S” by an occur-

rence of “T” is to appeal to a rule concerning sentences that merely have the same truth value but are not “logically equivalent” sentences, and hence to reduce the argument to (II). Yet that is exactly what Neale turns out to do, in an elaborately circuitous manner.

The underlying appeal to such a simple version of the argument is disguised by the introduction of “inference” principles and a lengthy discussion that purportedly justifies employing those principles. The complexity of his discussion is also due, in part, to the use of a complex variant of descriptions like “ $(\iota x)(x = a \ \& \ S)$ ” to guarantee that the description will be fulfilled, whether “S” is true or not, in order to present two versions of the argument. (2001, 170-74) We will see what is involved below. Yet, for all the discussion and complication, at the crucial place in both of his arguments he simply replaces “S” by “T,” like a proverbial slight of hand “switch.” This is obscured, on one variant, by a distracting discussion of a supposed consequence of taking “ $S \leftrightarrow T$ ” as a premise, along with what plays the role of (3\*). It is done, on a second variant, by a discussion of the premises of (I), and hence the stronger “S & T” in place of “ $S \leftrightarrow T$ ,” with his version of (3\*). I will simply deal with one argument, which he calls a “complete connective proof,” though we will note what goes on in the other.<sup>1</sup> To do so we will use his notation, with “ $\phi$ ” replacing “S,” “ $\psi$ ” in place of “T” and the sign “ $\textcircled{8}$ ” in front of a sentence to indicate a context for that sentence. Such a context is like “‘ $\phi$ ’ designates ...” or “the fact that- $\phi$  = the fact that-....” Taking the second reading of the operator, “ $\textcircled{8}\phi$ ” and “ $\textcircled{8}\psi$ ” are transcribed, respectively, as “the fact that- $\phi$  = the fact that- $\phi$ ” and “the fact that- $\phi$  = the fact that- $\psi$ .”

Consider then two argument patterns:

- |   |                                 |          |
|---|---------------------------------|----------|
| A | 1N. $\phi \leftrightarrow \psi$ | premise  |
|   | 2N. $\textcircled{8}\phi$       | premise. |

<sup>1</sup> What they amount to is easily seen in terms of versions that use class abstracts rather than descriptions. If you assume “ $\psi \ \& \ \phi$ ” (take “ $\psi$ ” and “ $\phi$ ” as true) then— $\{x \mid x \text{ is } F \ \& \ \psi\} = \{x \mid x \text{ is } F \ \& \ \phi\} = \{x \mid x \text{ is } F\}$ ; if you assume “ $\psi \leftrightarrow \phi$ ” then— $\{x \mid x \text{ is } F \ \& \ \psi\} = \{x \mid x \text{ is } F \ \& \ \phi\} = \{x \mid x \text{ is } F\}$ , when the materially equivalent statements are true, and  $\{x \mid x \text{ is } F \ \& \ \psi\} = \{x \mid x \text{ is } F \ \& \ \phi\} = \emptyset$ , when they are false. Neale mentions (Neale, 2001, 173) that he borrows the second from a 1950s variant of Quine’s (see, for example, Quine, 1961, 159).

The premises can be understood, *in a sense*, to specify the *truth-functional equivalence* of the statements, (1N), and that one of the relevant facts is “self-identical,” (2N). We then get:

$$3N. \textcircled{3} (a = (\iota x)((x = a \ \& \ \phi) \vee (x = b \ \& \ \neg \phi)))$$

from (2N) by the use of the principle allowing substitution of logical equivalences. *Given a Russellian treatment of definite descriptions*, and “a” and “b” as *singular terms*, (3N) results by replacement of a logical equivalence. For it is understood that the *apparent identity* context is “just shorthand” for the expanded Russellian version that results in the former being transcribed by an existential statement.<sup>2</sup> (2001, 173) That existential statement is logically equivalent to “ $\phi$ .” Then we get to step (4N):

$$4N. (\iota x)((x = a \ \& \ \phi) \vee (x = b \ \& \ \neg \phi)) = \\ (\iota x)((x = a \ \& \ \psi) \vee (x = b \ \& \ \neg \psi))$$

(4N) comes from (1N) and the use of the Russellian contextual definition for definite descriptions. Thus (4N) “is shorthand for”:

$$(\exists x)[((y)((y=a \ \& \ \phi) \vee (y=b \ \& \ \neg \phi)) \leftrightarrow y=x) \ \& \ (\exists z)((w)((w=a \ \& \ \psi) \vee (w=b \ \& \ \neg \psi)) \leftrightarrow w=z) \ \& \ x=z].$$

The next step is the crucial one. It takes us from (4N) and (3N) to

$$5N. \textcircled{3}(a = (\iota x)((x = a \ \& \ \psi) \vee (x = b \ \& \ \neg \psi))),$$

from which one goes directly, via the substitution of logical equivalences, to

$$6N. \textcircled{3}\psi.$$

Thus concludes one of Neale’s versions of The Great Fact argument, (Neale, 2001, 173-74).

<sup>2</sup> There are questions about such identity contexts that we bypass. Consider, for example, whether “ $x=y$ ” is defined by means of “ $(f)(fx \leftrightarrow fy)$ ,” as in *Principia*, or taken as primitive. See also (1950, 83-84).

The second version employs a less complex definite description and is the usual pattern of argument employed in presentations of the “Great Fact” argument. The usual description is simply “ $(\exists x) (x = a \ \& \ \phi)$ ” and is employed with the premises of the pattern (I).

B	1N*. $\phi$	premise
	2N*. $\psi$	premise
	3N*. $\exists\phi$	premise.

Here, the first two premises, *in a sense*, express that the statements are taken as true. (The point of the stressed qualifying “in a sense,” in speaking of both versions of the argument, will emerge.) Neale sets out both arguments since he takes it to be important that variant A allows for the definite descriptions to be uniquely satisfied if “ $\phi$ ” and “ $\psi$ ” have the same truth value—for “...we want to know ... whether  $\exists\phi$  leads to  $\exists\psi$  when  $\phi$  and  $\psi$  are materially equivalent, not just when they are both true.” (2001, 172) That has no real bearing on the philosophical issues connected with The Great Fact argument, but we will consider the issue mainly in terms of variant A.

(5N) supposedly follows by a “principle of substitution” that Neale employs, following Davidson’s and Quine’s use of such a principle, but Neale undertakes to provide an extended justification of the use of the principle. His justification purportedly shows that an objection that can be raised to the move to step (5N) in (A) is not viable. That objection, simply put, is that one gets nowhere if one employs expansions of the definite descriptive phrases, i. e. Russell’s “longhand” existentially quantified expressions, in place of the iota-operator “shorthand.” Hence, according to the objection, the argument depends on the misleading employment of shorthand expressions—misleading in that all one really does is replace one “materially” equivalent statement by another. Thus all the argument does is offer a version of (II) that is hidden by the use of definite descriptions. Neale argues that the objection is not viable since the move to step (5N) can be mirrored by a sequence of steps employing expansions of the definite descriptions along Russellian lines. *This, in essence, is what his book comes down to.* If his argument were cogent it would make for an interesting paper, if not a 254 page book. However, when we look at the principle of substitution Neale employs (2001, 160-61), and the claim that it is justified by there being a corresponding sequence of steps using expansions of the descrip-

tions, we find him shifting into “informal” English expositions, and not mirroring a Russellian *derivation*. Those carry the burden of justification for the move to (5N). They purportedly provide such a justification by repeatedly explaining that the key expressions with descriptions are “just shorthand for” (2001, 171) or “shorthand for” (2001, 173) Russellian-style expansions with existential quantifiers.

What this talk of “just shorthand” covers over is that Neale, to get to (5N), tacitly *assumes* that—in dealing with the expansions of “ $(\exists x)((x = a \ \& \ \phi) \vee (x = b \ \& \ \neg \phi))$ ” and “ $(\exists x)((x = a \ \& \ \psi) \vee (x = b \ \& \ \neg \psi))$ ,” as employed in (A), and, correspondingly, with those of “ $(\exists x)(x = a \ \& \ \phi)$ ” and “ $(\exists x)(x = a \ \& \ \psi)$ ” in (B)—he may rely on a far from innocent appeal to “ $\phi$ ” and “ $\psi$ ” *being truth-functionally equivalent*, in the one case, and both *being true*, in the other. This appeal justifies replacing the occurrence of “ $\phi$ ” in the expansion of the crucial sentence by “ $\psi$ .” Neale, of course, does not put it so simply—for had he done so he would have no foundation for the presentation in his book, or need to present it. Yet, when one thinks about it, it is obviously odd that an argument that carries through when one uses “just shorthand” is blocked when one uses “longhand.” One would think something is clearly wrong when one introduces rules to “bridge the gap,” as it were. What is wrong is what forces Neale to take his detour through the meta-language by “reasoning” informally to justify his use of shorthand. But the promised parallel argument in longhand is never offered—nor can such a “mirrored” sequence be constructed without, in one way or another, relying on a replacement that is justified by the relevant “material equivalence.” And that appeal to material equivalence is one Neale makes by way of his additional rules.

Recall the familiar, if not precise, idea that a statement P *logically entails* a statement Q if and only if Q is true in every “model” in which P is. Neale cannot construct a *proof* or *derivation* of The Great Fact’s conclusion or show that it is *logically entailed* by the premises in a straight forward manner. So what he does is appeal to a principle allowing replacement of *material equivalences* by *reasoning* as follows. *Given the premise  $\phi \leftrightarrow \psi$ , neglecting quotation marks, for the case of (A), we need consider only those models* in which  $\psi$  holds if and only if  $\phi$  does—i. e. in which the two have the same truth value. Alternatively, in the case of (B), we need consider only those models in which the two statements are true (and thus, trivially, are models in which they have the same truth value). In any *such model* the crucial statements employing the descriptions—or their existen-

tial transcriptions—making use of  $\psi$  will have the same truth value as corresponding statements involving  $\phi$ . Thus, in place of mirroring a sequence of steps employing expansions of definite descriptions along Russellian lines, as was promised, we are told that the key formula with  $\psi$  results from the one with  $\phi$  since it is true “if  $\phi$  and  $\psi$  are both true,” in the case of (B), and “whenever line (1) [ $\phi \leftrightarrow \psi$ ] is true” in the case of (A). (2001, 171-172)

In short, in shifting to such informal reasoning to justify his additional “rules,” Neale shifts to considering formulae with respect to *all models of a certain kind*. The appropriate kind of model, we can call them *equivalence models*, or “E-models,” are those where, for (A),  $\phi$  and  $\psi$  have the same truth value, and, for (B), where  $\phi$  and  $\psi$  are both true.<sup>3</sup> Thus he takes the biconditional (or a conjunction) being a premise to allow the shift to speaking, meta-linguistically, of  $\phi$  being true if and only if  $\psi$  is (or of both being true)—*and from there* to speaking only of the models where they have the same truth value, for (A), or in which both are true, for (B). The quick, informal meta-linguistic reasoning then serves to justify the *derivation* of one step from another in The Great Fact argument—hence justifying the move to (5N). With  $\mathbb{E}f(\phi)$  and  $\mathbb{E}f(\psi)$  being, respectively, the Russellian expansions of the contexts for the “operator”  $\mathbb{E}$  in (3N) and (5N), that amounts to holding that since  $\phi \leftrightarrow \psi$  and  $f(\phi)$  entail  $f(\psi)$ ,  $\mathbb{E}f(\phi)$  entails  $\mathbb{E}f(\psi)$ . [And, correspondingly, in the case of (B).] This supposedly allows a detour around the barrier blocking a straight forward *derivation*, a “*mirroring*” he cannot supply—the detour being provided by the commentary on  $\phi$  and  $\psi$  having the “same truth value” or “both being true,” since  $\phi \leftrightarrow \psi$  or  $\phi \ \& \ \psi$  is a premise. That is why we find the key phrases occurring at crucial points in his informal “reasoning” about his new rules— “if  $\phi$  and  $\psi$  are both true” (Neale, 2001, 171) and “... whenever line (1) [ $\phi \leftrightarrow \psi$ ] is true” (2001, 174). But all that he has done, when all is said and done, is, in a winding way, replace “ $\phi$ ” by “ $\psi$ ,” given their material equivalence.

One reason that Neale does not seem to realize that that is all that he actually does may be due to his dwelling on the familiar point that Russell and Whitehead take the contexts of *Principia* to be truth functional. (That is basically true and, in fact, was one reason for Russell’s developing his mul-

<sup>3</sup> The term “E-model” is, of course, not used by Neale. It is introduced to highlight just what it is that he does.



tuple relation analysis of intentional verbs.<sup>4</sup>) Neale thinks that not understanding this feature of *Principia* is what lies behind the objection to The Great Fact argument that is based on rejecting the move to step (5N). He simply fails to understand that the objection is based on arguments like his having to appeal, explicitly or implicitly, to a rule sanctioning replacement given material equivalence, which is what really permits the move to (5N). What is objected to is the attempt to avoid openly acknowledging that one makes use of *such* a rule of replacement, and hence pointlessly employs the pattern (N), a sibling of (II):

(N)	1N*. $\psi \leftrightarrow \phi$	premise
	2N*. $\textcircled{3}\psi$	premise
	3N*. $\textcircled{3}\phi$	conclusion

Using (N) openly removes the seeming “paradox” surrounding talk about facts, since (N) is as pointless as (II). Relying on E-models in his informal “reasoning” justifying the new rules he introduces, Neale somewhat disguises his appeal to material equivalence, i. e. to (N). Realizing that, one sees that the issue is not about definite descriptions nor about “positing entities” like facts, nor even about providing a “semantics for definite descriptions” that function “outside the realm of extensional logic.” (Neale, 2001, 223) It is simply about purporting to offer an argument that neither depends on nor implicitly makes use of *Principia*’s:  $p \leftrightarrow q \ . \ . \supset \ . \ f(p) \leftrightarrow f(q)$ . That was what seemed to motivate Davidson’s attempt to depend only on *logical equivalence* and substitution of “identicals.” The simple point is that the purported substitutions and replacements Neale introduces are not what they are claimed to be.

<sup>4</sup>The “extensionality” of the first edition of *Principia Mathematica* raises questions, given the use of functions and the taking of *diverse* functions of different order to have *identical* extensions. Thus, there is a sense in which *Principia*’s functions are not extensional, as the term “extensional” is used. Thus, in the second edition, Russell and Whitehead comment on the distinction between classes and functions in the first edition. (1950, xxxix, 83-84) There are also questions about functions themselves and the distinction between functions, on the one hand, and properties and relations (as constituents of atomic facts), on the other. Functions, i. e. propositional functions, are clearly not such constituents, and, if one considers the ontological foundation on which *Principia* rests to be composed of particulars and universal properties and relations formed into atomic facts, there are grounds for holding that such functions, along with propositions, disappear as “incomplete symbols.” We will return to this matter below in considering Neale’s discussion of incomplete symbols.

Given Neale's citation, at various places, of *Principia* theorems involving descriptive phrases and his reiteration that *Principia* is an extensional system, it is surprising that he does not simply cite the *Principia* passage—

All the functions of propositions with which we shall be specially concerned will be truth-functions, i. e. we shall have

$$p \leftrightarrow q \ . \supset \ . \ f(p) \leftrightarrow f(q). \quad (1950, 115)$$

—and end the discussion there, rather than belaboring *Principia*'s extensionality and emphasizing:  $(\iota x) \phi x = (\iota x) \psi x \ . \supset \ : \ \chi \{(\iota x) \phi x\} \leftrightarrow \chi \{(\iota x) \psi x\}$ . (2001, 160) For with “ $\textcircled{3}p$ ” covered by “ $f(p)$ ” the matter is settled, simply, by (N). With all his rules, Neale sometimes seems to say just about that, which leaves one to wonder why he bothers doing what he does. [Regarding *Principia*'s extensionality one might note the phrase “specially concerned” in the above, as well as the discussion of “extensional” contexts in the commentary in *Principia*. The matter is complicated by the question of diverse functions with the same extension, the first edition's Axiom of Reducibility and some apparent “identity” contexts (1950, 83-84).]

## II. Facts and Possible Facts: Representing and Describing

Russell can be seen as having introduced a way of denoting facts by definite descriptions in his first presentation of the theory of descriptions in “On Denoting.” There, speaking of complexes such as “the holding of the relation R between a and b,” he was concerned with the problem posed by false sentences, “aRb” say, when a does not stand in R to b. From a philosophical point of view, the problem such sentences pose is far more important than his concern with a denoting expression, such as “the King of France (in 1905),” apparently denoting the non-existent King of France. In his 1913 manuscript “Theory of Knowledge,” he developed a suggestion made in “On Denoting”:

- (T) “a is different from b” is true  $\leftrightarrow$  the difference between a and b subsists.

The idea was apparently to use a definite description so that one specified the truth ground for a relational sentence by employing a definite description. Thus one did not have to recognize a denoted possibility if “a is different from b” were false. The complex (fact) that was “denoted,” *not represented*, simply did not exist, and (T) remained true, in the later fashion of Convention-T. Put in pseudo Russellian-style symbols, with “p” as a variable ranging over (atomic) facts or “complexes” and ignoring that facts “subsist” or not, rather than exist or not, we have:

(RT) “aRb” is true  $\leftrightarrow$  E! ( $\exists$ p)(p is a standing in R to b).

Thus, quite unlike a Carnapian-style semantic designation rule, “L-rule,” linking names and predicates to objects and properties—“a” designates a, “b” designates Bismarck, “R” designates the relation of being taller than, etc.—it is suggested that atomic sentences are semantically linked to truth grounds by the familiar truth-functional biconditional, as in (RT). Thus, (RT) can be taken to be a semantic rule of the system, and hence among the “logical truths” or L-truths, in Carnap’s sense. Such a pattern has to be elaborated, but only one aspect of that need be of concern here. Taking (RT) as a semantic rule should specify not only the “truth-maker” of “aRb” but the connection between the juxtapositional pattern of the atomic sentence and the sentence used to link it to its truth ground. Thus what is required is really a tri-partite biconditional:

(RT\*) “aRb” is true  $\leftrightarrow$  aRb  $\leftrightarrow$  E! ( $\exists$ p)(p is a standing in R to b).

But there is a more basic problem in the 1905 suggestion, which, interestingly reappears in a one of Neale’s discussions, as we will see below. That problem is the use of the phrase “a standing in R to b.” For it appears that Russell really hasn’t avoided the use of atomic sentential patterns in a representational role. That use is simply shifted to the clause in the right side of “the” biconditional.

Russell recognized this by the time of the 1913 manuscript that he did not complete. (Russell, 1984; Hochberg, 2000) There he developed a way of avoiding such a use of a sentential phrase by employing a pattern along the following line. Let “C” represent a relation of containment between a fact (complex) and a component of the fact. Then one can, ignoring the complexity introduced by the need to *order* the terms—to distinguish the

fact that-aRb from the fact that-bRa—consider “the” fact to be described by:  $(\iota p)(pCa \ \& \ pCb \ \& \ pCR)$ .<sup>5</sup> We can now reformulate (RT\*) in terms of:

$$(RT^{**}) \text{ “aRb” is true } \leftrightarrow aRb \leftrightarrow E! (\iota p)(pCa \ \& \ pCb \ \& \ pCR).$$

What is relevant here is what now happens to The Great Fact argument on this adaptation of Russell’s pattern that was set out in the first decades of the 20<sup>th</sup> century.

For simplicity we reformulate the argument pattern using two monadic atomic sentences, “Fa” and “Gb,” as premises along with the two relevant L-true semantic rules, since the latter are “rules” or L-truths of the “system”:

- (1) Fa
- (2) Gb
- (3) “Fa” is true  $\leftrightarrow Fa \leftrightarrow E! (\iota p)(C(p, a) \ \& \ C(p, F))$
- (4) “Gb” is true  $\leftrightarrow Gb \leftrightarrow E! (\iota p)(C(p, b) \ \& \ C(p, G))$ .

We then arrive very quickly at

$$(5) \text{ “Fa” is true } \leftrightarrow Gb \leftrightarrow E! (\iota p)(C(p, b) \ \& \ C(p, G)),$$

since we have (1) and (2), and hence “Fa  $\leftrightarrow$  Gb.” But that is harmless and utterly trivial. We do not, and cannot, arrive at the conclusion that (5) *is a semantic rule giving the truth ground* for “Fa.” Nor can we derive “ $(\iota p)(C(p, a) \ \& \ C(p, F)) = (\iota p)(C(p, b) \ \& \ C(p, G))$ .” That can be seen to reflect the non-extensionality of contexts employing (explicitly or implicitly) notions like “designates,” or “semantic rule,” that involve, as in (II), sentences and “names” of sentences. Introducing descriptions like “ $(\iota x)(x = a)$ ” will not lead anywhere.<sup>6</sup> The Great Fact can be left to rest undisturbed.

<sup>5</sup> Resolving the questions about *order* in relational facts was a major problem Russell sought to resolve in the 1913 manuscript. Essentially what he did was have, in place of C, two relations for each standard non-symmetric dyadic relation, R. Each such relation held between a term and the fact and expressed both the “position” (initial term) of the constituent term in the fact and the normal relational content (left-of, for example).

<sup>6</sup> There are questions about the use of “=,” defined in Russell’s way, with descriptions of facts and with descriptions generally, but we can safely ignore those here. Simply take identity or diversity as primitive.

Russell's introduction of descriptions of facts, as above, was designed to avoid the introduction of non-existent or possible facts—as being represented or designated by false atomic sentences—in linking such sentences to truth grounds. Interestingly, nowhere in his book does Neale take up that problem for a correspondence theory, though he does mention it in passing. “But the representational power of false beliefs, utterances and inscriptions cannot be neglected, and so theories of facts have been supplemented (or supplanted) by theories of states of affairs, and propositions.” (Neale, 2001, 3) Since he takes the pattern of The Great Fact to be applicable to propositions (The Great Proposition), beliefs (The Great Belief) and, presumably states of affairs, if such theories adopt corresponding principles of substitution to those he has laid down (2001, 202-203), such cousins of The Great Fact can be ignored. What is worth noting is that his preoccupation with The Great Fact (Belief, Proposition) is possibly responsible for his ignoring an important aspect of a real problem facing attempts to deal with facts. It is a problem that both Russell and Wittgenstein were well aware of, given their familiarity with the Austrian philosophical tradition. Just how oblivious Neale is to the problem is indicated by his discussion of B. van Fraassen's uses of a sign pattern like “ $\langle a, F \rangle$ ” for—the complex *that*-Fa—and of “ $\{ \langle a, F \rangle \}$ ” for—the fact that “makes ‘a is F’ true.” (Neale, 2001, 87) Neale goes on to say that “If we are Russellians, we can think of “ $\{ \langle a, F \rangle \}$ ” as a definite description of a fact—‘the fact that a is F’—though not a name of that fact.” (2001, 87) Though speaking of being “Russellians” he doesn't mention Russell's writing, about an arbitrary n-term atomic relational complex: “The actual complex  $\gamma$  itself, whose existence is affirmed by description in our associated molecular complex, cannot be directly named, ... but a complex name for it must be descriptive.” (Russell, 1984, 148).<sup>7</sup> In fact Russell's 1913 manuscript, published in 1984, does not appear in Neale's eighteen page list of references. What is also odd is that Neale, in using the expression “the fact that a is F,” makes the same key mistake that Russell made in 1905. Russell explicitly corrected it, for relational contexts, in 1913, given his preoccupation with resolving the problem of relational order.

The problem Neale ignores not only appears with the use of the clause “that a is F” in such a description, but in van Fraassen's explicitly

<sup>7</sup> Russell focused on relational contexts due to his concern with the problem of order that he thought, in 1913, required further analysis of facts with relations. (Russell, 1984, 148)

acknowledging a *complex* like  $\langle a, F \rangle$ , as seemingly embedded in “the fact”  $\{\langle a, F \rangle\}$ . One obviously has not employed a description in order to avoid recognizing a non-actual fact—for one implicitly has acknowledged what is represented by the “that” clause in Neale’s description—or explicitly recognized a “complex” that is not a “fact.” Russell’s attempt, as early as 1905, to use descriptions to avoid recognizing non-obtaining states of affairs (non-actual complexes) is completely overlooked.<sup>8</sup>

Suppose in place of (RT\*\*) one followed Carnap directly:

(CD) “Fa” designates the state of affairs that a is F.

(CT) “Fa” is true  $\leftrightarrow$  the state of affairs that a is F obtains.

Given that (CD) is an L-True rule of the system, it is true whether or not the state of affairs that a is F does obtain. Hence Carnap has a correspondent of the atomic sentence irrespective of the latter’s truth value—he thus has possible (non-actual, non-obtaining) states of affairs (or facts, if one chooses) and existent (actual, obtaining) states of affairs, or facts. Whether or not one calls such possible facts “propositions” is of no matter here—what is of concern is whether one introduces such “entities.” As early as 1905, Russell sought to avoid the pattern (CD) introduces and the type of entity the pattern brings with it. His theory of descriptions allowed him to do so and thus resolved the long ignored “third puzzle” that theory was said to remove in “On Denoting.”

The connection of descriptions to the issue about possible facts is even touched on in *Principia*:

The universe consists of objects having various qualities and standing in various relations. Some of the objects which occur in the universe are complex. When an object is complex, it consists of interrelated parts. Let us consider a complex object composed of two parts *a* and *b* standing to each other in the relation *R*. The complex object “*a*-in-the-relation-*R*-to-*b*” may be capable of being *perceived*; when perceived, it is perceived as one object.... Since an object of perception cannot be nothing, we cannot perceive “*a*-in-the-relation-*R*-to-*b*” unless *a* is in the relation *R* to *b*. Hence a judgment of perception, according to the above definition, must be true. ... In fact, we may define *truth*, for where such

<sup>8</sup> One might think that the problem Neale fails to see also arises in connection with a description like “ $(\exists p)(C(p, a) \ \& \ C(p, F))$ .” For, while that does not contain “Fa,” it does contain sentential patterns, such as “ $C(p, a)$ .” For a discussion about why it does not and why the pattern avoids the “Bradley regress” see Hochberg (2000).

judgments are concerned, as consisting in the fact that there is a complex *corresponding* to the discursive thought which is the judgment. That is, when we judge “*a* has the relation *R* to *b*,” our judgment is said to be *true* when there is a complex “*a*-in-the-relation-*R*-to-*b*,” and is said to be *false* when this is not the case. (1950, 43)

We will give the name of “a *complex*” to any such object as “*a* in the relation *R* to *b*” or “*a* having the quality *q*,” or “*a* and *b* and *c* standing in the relation *S*.” Broadly speaking, a complex is anything which occurs in the universe and is not simple. We will call a judgment elementary when it merely asserts such things as “*a* has the relation *R* to *b*,” “*a* has the quality *q*” or “*a* and *b* and *c* stand in the relation *S*.” Then an *elementary* judgment is true when there is a corresponding complex, and false when there is no corresponding complex. (1950, 44).

The implicit employment of the theory of descriptions to resolve the problem of the purported representation of possible, but not actual, states of affairs is apparent. The singling out of the specification of truth grounds for elementary sentences also leads directly to Russell’s holding that the “definition of truth” is different in the case of elementary judgments—elementary *truth*—from that of non-elementary judgments. Thus the correspondence theory of truth—with atomic facts as truth grounds for elementary judgments—that Russell developed did not comprise a univocal truth predicate, *irrespective* of the need to deal with the familiar semantic paradoxes. This was to be a persistent theme in Russell’s thought.

Contemporary “correspondence theorists” have generally avoided or skirted the problem posed by implicitly taking atomic sentences to represent possible facts. (D. M. Armstrong is one, for example, who simply ignores the problem.) Neale, likewise, does not address it, for he has other interests such as pointing to supposed constraints on identity conditions for facts. The constraints turn out to be dictated by his substitution rules and the conversion rules for the iota operator. What this amounts to is his claim that those who would speak of facts cannot allow for such rules, since The Great Fact will appear (along with its cousins for propositions, beliefs, etc.). Yet, problematically, those who would speak of facts apparently must allow for them, since Neale takes himself to have shown that the additional rules merely embody innocent notational shorthand. But the threat is empty and much of the extensive discussion generated as a consequence, including criticisms of others written, in part, in response to objections raised by his earlier paper, pointless.

Neale’s ignoring of the connection between the introduction of a linguistic apparatus for speaking about or denoting facts and the real prob-

blems faced by theories of facts highlights the irrelevance of his discussion to real philosophical problems. A correspondence-type theory, of the kind Neale considers, involves two steps in taking facts as truth grounds for atomic sentences.<sup>9</sup> First, one has to link the atomic statement to its “ground of truth,” as in:

- (1) “Fa” designates the state of affairs that-a is F,

where “a” is a logically proper name and “F” a primitive predicate. Second, one has to specify that, for “Fa” to be true, the state of affairs must obtain, thus we have:

- (2) “Fa” is true  $\leftrightarrow$ . “Fa” designates the state of affairs that-a is F & a is F (the state of affairs obtains).

This is what raises the problem, for such a correspondence account, of recognizing non-obtaining states of affairs (possible facts), since (1) is true, and, moreover, L-true as a semantical rule, irrespective of whether “Fa” is true. That aside, one cannot simply replace “that-a is F” in (1) by “that-b is G,” where “Gb  $\leftrightarrow$  Fa” (or simply where both are true). In that sense, (1), given the role of the semantical term “designates,” is not an extensional context. However, if the system allows the redundancy of diverse “logically proper names” for the same particular and diverse primitive predicates designating the same property so that, say, “a=b” and “F=G” are L-truths, one can, if not restricted by one’s rules governing “designates,” then derive:

- (3) “Fa” designates the state of affairs that-b is G,  
 (4) “Fa” is true  $\leftrightarrow$ . “Fa” designates the state of affairs that-b is G & b is G (the state of affairs obtains).

<sup>9</sup> There is a question regarding whether there are facts that are not atomic. Recall the issues about negative and general facts that Russell, among others, discussed. To emphasize an important aspect of Neale’s flawed line of argument it is simplest to focus on atomic facts. For, if The Great Fact argument has any worth, one should be able to “switch” any true atomic statement with any other via the manipulations of the pattern. As Neale notes, it “would be every bit as devastating” to produce The Great Atomic Fact (2001, 130).



But, this is as it should be and has nothing to do with purportedly deriving a pattern with a definite description. For, suppose “ $a = (\exists x)Hx$ .” That does not lead to:

(5) “ $Fa$ ” designates the state of affairs that  $(\exists x)Hx$  is  $G$ .

And, clearly, it should not do so in virtue of issues about purported “existential states of affairs”—whether one requires such, their difference from atomic states of affairs, etc. All this touches on serious questions faced by those who advocate the existence of facts (and truth-makers generally).

### *III. Descriptions, Classes, and Propositions as Incomplete Symbols*

Neale writes:

Russell certainly deployed his Theory of Descriptions to ontological benefit, but it was not an integral feature of the theory that certain types of entity could be defined away contextually. The chief ontological benefit for Russell was that it allowed him to treat certain sentences as true or false without seeing their grammatical subjects (*or* grammatical direct objects, for that matter) as standing for things that don’t exist, an idea he rightly came to regard with repugnance. This gave him a basic ontology of particulars, universals, and facts. (2001, 119)

While he is certainly correct to observe that Russell clearly sought to show that certain expressions were not, logically, functioning as “grammatical subjects,” his contrasting of “grammatical” with “ontological” distorts the roles the theory of descriptions and the notion of an incomplete symbol played in Russell’s classic attempts at *ontological analysis* and *reduction*. Recall the obvious use of the theory of descriptions, noted above, to avoid *reference to* “possible” facts—non-existent states of affairs—in specifying the truth grounds for elementary sentences—a use Neale ignores. It is precisely, the *ontological* import of the theory of descriptions that is important, for Russell and subsequent philosophers in the analytic tradition. That import would be hard to exaggerate and goes well beyond a concern with grammatical form and apparent grammatical subjects.

In the 1903 *The Principles of Mathematics*, Russell wrote: “And the contradiction discussed in Chapter x. proves that something is amiss, but what this is I have hitherto failed to discover.” Shortly after, in 1905, as he cryptically noted in *My Philosophical Development* and in the first volume

of the autobiography, his theory of descriptions led him to the solution: “It was clear to me that I could not get on without solving the contradictions, and I was determined that no difficulty should turn me aside from the completion of *Principia Mathematica* ... [I]n the spring of 1905 ... I discovered my theory of descriptions, which was the first step towards overcoming the difficulties which had baffled me for so long.” (1967, 243) He did not explain there how the theory of descriptions played the role of a “first step,” but what he meant was probably: first, that expressions like “the property of not being a self-applicable property,”  $(\iota\phi)(\psi)(\phi\psi \leftrightarrow \neg \psi\psi)$ , and “the class of all classes that are not members of themselves,”  $\{\alpha \mid \beta \in \alpha \leftrightarrow \beta \notin \beta\}$ , are not to be taken as referential expressions that *represent* a property and a class, respectively, but as contextually defined signs—incomplete symbols—that cannot be employed in purported instantiations to yield a paradox.<sup>10</sup> Second, the theory of descriptions pointed the way to the “no-class” theory of classes, and, derivatively, that the *Russell property* did not exist. Shortly thereafter the latter point would be codified in a theory of logical types.

What is clear is that Russell often uses the phrase “incomplete symbol” in two senses—on one, sign patterns, a class expression for example, is taken as an “incomplete symbol”—on the other, the entities such expressions are purportedly used to represent, the class (or the kind, classes) are called “incomplete symbols.” The second use clearly places the emphasis on his having “deployed” the theory of descriptions for “ontological benefit.” For it is the heart of the no-class theory of the first edition of *Principia Mathematica*. This is not really a matter of secondary “deployment” but a core idea behind the removal of classes as objects, along with the natural numbers, since an interpretation of the Peano-Dedekind postulates can be constructed in terms of class abstracts in *Principia*. One can easily see the heart of Russell’s logicism to lie in the claim that, since arithmetical truths are construed as logical truths and arithmetical concepts are construed in terms of logical concepts, one need not recognize either classes or numbers as entities. The theory of descriptions becomes an ontological scalpel that effectively functions as a variant of Occam’s Razor in a variety of contexts.

Neale seems to think otherwise since Russell’s elimination of classes eliminated a “category” of entity while his analysis of descriptions in con-

<sup>10</sup> He linked the theory of descriptions to the “no-class” construal of class expressions, treating the latter along the lines of definite descriptions, in various writings of the first three decades of the 20<sup>th</sup> century, while taking that analysis of such expressions as instrumental to resolving “the paradox.”

nection with “objects” (particulars) did not. But in addition to the case of classes, Russell employed descriptions to avoid possible facts (non-existent states of affairs), though he did not dispose of the category of facts, and he used the notion of an incomplete symbol to dispense with propositions as entities since:

...the phrase which expresses a proposition is what we call an “incomplete” symbol; it does not have meaning in itself, but requires some supplementation in order to acquire a complete meaning. This fact is somewhat concealed by the circumstance that judgment in itself supplies a sufficient supplement, and that judgment in itself makes no *verbal* addition to the proposition. Thus “the proposition ‘Socrates is human’” uses “Socrates is human” in a way which requires a supplement of some kind before it acquires a complete meaning; but when I judge “Socrates is human,” the meaning is completed by the act of judging, and we no longer have an incomplete symbol. The fact that propositions are “incomplete symbols” is important philosophically, and is relevant at certain points in symbolic logic. (1950, 44)

But what is important is not that Russell used descriptions for particulars, while not dispensing with all particulars as entities. Rather, it is his attempt to avoid the need for recognizing classes (and numbers), possible (but not actual) facts, and propositions, *along with* non-existent particulars. Besides overlooking the fact that class abstracts are a form of definite description (which plays a role in some alternative formulations of The Great Fact), Neale’s point about not using descriptions to dispense with all particulars is quite trivial. It is interesting though that Russell later, in 1940 and 1948, dismissed basic particulars (spatial points, temporal moments, substrata) and construed ordinary particulars as complexes of universal qualities. Unfortunately, he misleadingly called such qualities, like *red*, “particulars.” He did so because they were no longer functioning as entities that are “attributed” but were now *constituent elements* of certain complexes of qualities. And, in the 1940s, such complexes of *compresent* qualities, taken as *complex qualities*, were of the same *logical category* as qualities like *red*—of the *same logical type*. That peculiar use of “particular” aside, particulars were removed from Russell’s world via the mechanism of logical analysis as a tool of *ontological* analysis and reduction.

In *Principia* class abstracts parallel the contextual definitions for definite descriptions with

$$*20.01 \quad f\{\hat{u}(\psi u)\} \quad . =: (\exists \phi) : \phi!x . \leftrightarrow_x . \psi x : f\{\phi! \hat{u}\} \text{ Df.}$$

and

The condition corresponding to

$$E!(\iota x)(\psi x) \text{ is } (\exists \phi) : \phi ! x . \leftrightarrow_x . \psi x,$$

which is always satisfied because of \*12.1.” (1950, 188).

The condition, transcribing “E!” via “exists,” can clearly be seen as emphasizing the non-existence of classes, since the apparent statement that a class exists simply states that there is a function equivalent to the “defining” function. What Russell says about “the existence of classes” is of further interest. The opening sentence introducing the discussion of classes in Section C of *Principia* is: “The following theory of classes, although it provides a notation to represent them, avoids the assumption that there are such things as classes.” (1950, 187) One avoids such an assumption due to the contextual definitions for class expressions “just as, in \*14, we defined propositions containing descriptions.” This is the idea of the “no-class” theory. It leads to:

When a class  $\alpha$  is not null, so that it has one or more members, it is said to exist. (This sense of “existence” must not be confused with that defined in \*14: 02 .) We write “ $\exists ! \alpha$ ” for “ $\alpha$  exists.” The definition is

$$*24:03 \exists ! \alpha = (\exists x)(x \varepsilon \alpha) \quad \text{Df. (1950, 216)}$$

The reference to \*14: 02 is to the contextual definition of “E!” for definite descriptions. This idea had already been expressed in *The Principles*: “Another very important notion is what is called the *existence* of a class—a word which must not be supposed to mean what existence means in philosophy. A class is said to exist when it has at least one term.” (1956, 21) The separation of “what existence means in philosophy” from the ascription of existence to a class in the theory of classes is worth noting, given Russell’s supposed focus on “grammar” as opposed to ontology.

As mentioned earlier, classes and class expressions, as well as definite descriptions and non-existent denotations, are all characterized as “incomplete symbols.” In the case of the signs this means that they are eliminable by contextual definition—they disappear in the “analyzed” statements that replace those that contain them in their contextual use—and do not have a meaning on their own. In the case of a “purported” represented

object, it would mean that the supposed entities are not taken as existents—entities that are presumed to exist by the employment of the linguistic system. Given Russell’s no-class theory, the *Principia* analogues of the primitive terms of the Peano-Dedekind axioms, being explicitly defined in terms of class signs and the remainder of the logical apparatus, are not taken to represent entities—either classes or any other purported “objects.” Russell provided a summary statement of these themes in a 1924 essay:

This definition ... avoids the inference to a set of entities called ‘cardinal numbers’, which were never needed except for the purpose of making arithmetic intelligible, and are now no longer needed for that purpose.

Perhaps even more important is the fact that classes themselves can be dispensed with by similar methods. ... all the propositions in which classes appear to be mentioned can be interpreted without supposing that there are classes. (1971, 327)

The emphasis on ontological import couldn’t be clearer, and the theory of descriptions would be a major device for the elimination and avoidance of purported problematic entities, and hence of philosophical problems, for Russell and for those who followed in the development of the analytic tradition.

Russell characterizes incomplete symbols in slightly different phrasings as:

- (a) By an “incomplete” symbol we mean any symbol which is not supposed to have any meaning in isolation, but is only defined in certain contexts. (1950, 66)
- (b) The symbols for classes, like those for descriptions, are, in our system, incomplete symbols: their *uses* are defined but they themselves are not assumed to mean anything at all. That is to say, the uses of such symbols are so defined that, when the *definiens* is substituted for the *definiendum*, there no longer remains any symbol which could be supposed to represent a class. (1950, 71, 72)
- (c) That is to say, the phrase which expresses a proposition is what we call an “incomplete” symbol; it does not have meaning in itself but requires some supplementation in order to acquire a complete meaning. .... The fact that propositions are “incomplete symbols” is important philosophically .... (1950, 44)

Note, in (c), his speaking of both a proposition and “the phrase which expresses” it as “incomplete symbols.” Overlooking such passages, Neale finds Russell’s discussions likely to be incapable of consistent interpretation and proceeds to “disentangle” the “quite distinct ideas” of being “incomplete,” “contextual definition” and “disappearing on analysis.” He pro-

ceeds to explain that “quantified noun phrases” and “connectives” are incomplete symbols, and yet some of them do not disappear on analysis. (Neale, 2001, 224) But such symbols *are not incomplete symbols*, on Russell’s use, as he explicitly states, though there is a familiar sense in which “syncategorematic” expressions are frequently spoken of as “incomplete” or having meaning only in context. Yet familiar senses are not of concern here. For Russell, first, an *incomplete symbol*, in the sense in which that phrase is applied to *signs* or *sign patterns*, is a *defined* expression and, moreover, defined in such a way that one can say, as Russell does above in (b), that “their uses are defined”—i.e. their employment in sentential patterns. But neither the existential quantifier, nor the universal quantifier, nor the function signs for negation and disjunction, are defined signs in *Principia*—syncategorematic though they may be. Both quantifier expressions are taken to “embody” primitive ideas and the connectives for disjunction and negation are clearly symbols “for” (represent) the two truth functions. [A familiar definition of one quantifier sign, by means of the other, is also discussed (1950,130-131, 138-139).]

The statement (1) is symbolized by “ $(x). \phi x$ ,” and (2) is symbolized by “ $(\exists x). \phi x$ .” No definition is given of these two symbols, which accordingly embody two new primitive ideas in our system. (1950, 15) They are (1) the Contradictory Function, (2) the Logical Sum, or Disjunctive Function, (3) the Logical Product, or Conjunctive Function, (4) the Implicative Function. ... Simplicity of primitive ideas and symmetry of treatment seem to be gained by taking the first two functions as primitive ideas. (1950, 6)

Mixing Russell’s use of “incomplete symbol” with the notion of “syncategorematic” may be due to reading the OED, a philosophical gospel to some, which specifies the meaning of the latter, in one example, by: “Russell’s contextual or syncategorematic definition of definite descriptions is equivalent to the conjunction of three propositions, one of which embodies a uniqueness claim.” (OED, 2<sup>nd</sup> ed., 1989) But Russell, by expressly taking the two truth functional expressions and the quantifiers (or at least one) as *primitive* in *Principia*, does not take such logical signs, grammatically syncategorematic as they may be, to be either contextually defined or *incomplete symbols*, on his use of the latter phrase for “signs.” In fact, the two pairs of signs are linked with problems, unresolved for him at that time—the status of logical forms, and of propositional functions. He attempted to address those in the 1913 manuscript that would remain unpublished until 1984.

Neale cites, in passing, an interesting passage from *Principia* that is meant to illustrate an obvious difference between the use of “the” in the sentence “The present King of France is bald” and in the phrase “the present King of France”:

Thus all phrases (other than propositions) containing the word *the* (in the singular) are incomplete symbols: they have a meaning in use, but not in isolation. (1950, 67)

What he doesn’t note is an interesting question that arises from juxtaposing that passage with one cited earlier:

Thus “the proposition ‘Socrates is human’” uses “Socrates is human” in a way which requires a supplement of some kind before it acquires a complete meaning; but when I judge “Socrates is human,” the meaning is completed by the act of judging, and we no longer have an incomplete symbol. The fact that propositions are “incomplete symbols” is important philosophically, and is relevant at certain points in symbolic logic. (1950, 44)

It is easy to be misled here. Russell is not just focusing on the use of the term “the” in the phrase “the proposition ‘Socrates is human’.” It is the far more interesting claim that is explicit in his saying “but when I judge ‘Socrates is human’.” When one utters (writes) a token of the sentence it will be in a context of judging, considering, believing, etc. The sentence serves to express the content of what is judged, considered, etc. Shifting to the relational case that Russell explicitly considers in his 1913 manuscript, one sees how the analysis fits into his multiple relation theory of judgment of that period. For the fact that Russell judges that Socrates is wiser than Plato is the truth maker for the further four-term propositional expression:

Judges (Russell, Socrates, wiser, Plato).

The “proposition,” in terms of the 1913 manuscript, can be taken as the function (expressed by)

$\Psi(x, \text{Socrates, wiser, Plato, } \delta)^{11}$

<sup>11</sup> This is also put in the form of an existentially quantified sentence: “ $(\exists \Psi)(\exists x)\Psi(x, \text{Socrates, wiser, Plato, } \delta)$ ,” where “ $\delta$ ” represents the logical form, omitted above—in this case that of “dual complexes”— $\psi xy$ . (Russell, 1984, 114-115) Doing that, Russell also speaks of the proposition as a “form” and as an existential fact, not a function:

that, for the argument pair <judges, Russell>, yields the judgment *fact* (sometimes itself called a “proposition”) as value. Thus, for Russell, propositions, taken as such functions, are dispensed with as “incomplete symbols”—in the sense of that phrase that applies to purported entities—along with classes and non-existent kings. Alternatively, they are taken as existential facts or “forms,” but not logical forms. Taken in any of these ways they “disappear” as they are not constituents of the resulting judgment fact, which is why Russell calls them “incomplete symbols.” While sentences, *propositions* in one sense of that term, like “Socrates is human,” are neither “defined” nor syncategorematic, purported propositional entities are rejected, as *incomplete symbols*, since they are not required as constituents of judgment facts. To speak of such purported entities as syncategorematic would be to speak nonsense.

The simple point is that the ontological import of the theory of descriptions, along with Russell’s notion of an “incomplete symbol,” is not adequately grasped if one focuses on the *linguistic roles* of subject terms, of syncategorematic expressions and of eliminable defined expressions. Clearly not all sentences are eliminated or sensibly spoken of as contextually defined—but “propositions,” as entities, are purportedly eliminated, as “incomplete symbols,” along with “possible” facts, the latter via definite descriptions of complexes. Seeing what Russell is about is helped if one is not overly eager to reiterate how confused he supposedly was about “use” and “mention,” and, amusingly, “contextual” and “stipulative” definitions.

Regarding Russell’s supposed confusion. Consider, first, our ordinary arithmetical “system” and use of natural numbers, addition, and multiplication. Consider, second, a more or less formal axiomatic system—the Peano postulates—with the signs “0”, “N” and “S” as primitive signs—either as embedded in a first-order system of predicate logic or simply to be “reasoned about” in ordinary terms. Finally, third, we can consider the *Principia* system with *correlates* of the Peano primitive terms introduced by “definition.” Thus we have three diverse sets of signs—carrying the “notions” of zero, number and successor. One now thinks, typically, of what Russell did as using the third set to resolve certain philosophical problems that can be raised about the first set. As part of the resolution of those problems, he derived correlates of the postulates of the Peano sys-

“A given proposition will then be the fact ....” (1984, 177) See Hochberg (2000) for a discussion of these matters.



tem, as theorems, in *Principia*, using his “contextual” (explicit) definitions of the correlated terms. This involves distinguishing the three sets of terms carefully and noting that Russell did not define the primitive terms of Peano (that would make no sense) or the terms of “ordinary language” arithmetic. Correspondingly, we can talk about the occurrence of definite descriptive phrases in ordinary language and of their correlated expressions in a sketch of a *Principia*-type formal system—where the correlated terms, not the ordinary language expressions, are contextually defined. One also then notes the commentary, in ordinary language, about both expressions, the puzzles that are raised about those occurring in the ordinary context and the proposed resolution of the puzzles by appealing to certain contextual definitions in a formal schema.

All of this has been familiar for virtually a century in the talk of ordinary language, formal languages, and ordinary language commentary about both. It is standard fare in undergraduate courses in analytic philosophy, philosophy of language and, sometimes, beginning logic. There is nothing wrong with presenting a version of such themes in a purported scholarly book. That is done often enough. What is tedious is its being presented in a lecturing style that dwells on Russell’s purported confusions, but neither clarifies nor edifies, while pronouncing unhelpful conclusions: “A contextual definition does not seem to be exactly a stipulative or an explicative definition.” (2001, 227) Neale’s talk of “stipulative” and “explicative” simply mixes and stirs around questions about (i) the formal definitions of *Principia* signs, (ii) the connection of those defined signs with expressions and uses of ordinary language (or those of another system, of Peano arithmetic, for example), (iii) the “meta” discourse about both and (iv) the connections to the relevant philosophical problems. Russell, in his pioneering efforts, pointed the way, and subsequent developments, by Carnap, Bergmann and many others, sharpening distinctions (many suggested by Russell) regarding language about objects, language about language, etc., established various themes that do not require reiteration in order to repeat, yet again, long familiar criticisms of Russell—especially in a way that is neither clear nor incisive.<sup>12</sup>

<sup>12</sup> Neale cites D. Kaplan (1972) in his discussion of “stipulative” and “explicative.” On the issues behind such a distinction as applied to descriptions and “numerals” see Hochberg (1956, 1957, 1970, 1970a). Bergmann’s systematic discussions in the 1950s about philosophical uses of language, ideal languages and ordinary discourse about formal schemata will come to mind to anyone familiar with them. (Bergmann, 1959, for example) One who dwells on Russell’s supposed confusion about such matters

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should read his comments on the elimination of "numbers," as in (Russell, 1952, 209-210, 212).

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*Carlos A. Dufour*

**Inhärenz**

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Wie reagiert man auf eine philosophische Frage? Wenn sie auf einer Verwirrung der Sprache und Logik beruht, kann man diese Frage durch geeignete Analysen auflösen. Andernfalls mag ein zusätzliches Wissen, etwa aus der Mathematik oder der Biologie, die richtige Antwort aufzeigen. Zuletzt gibt es ehrgeizige Fragen, denen nur durch philosophische Theoriebildung zu begegnen ist.

Ausgehend von dieser Trichotomie und in Auseinandersetzung mit der gängigen Sprachanalyse vertritt der Autor die These, daß die Philosophie letztlich theoretisch sein muß und ontologisch sein kann. Dazu ist es nötig, verbreitete Vorurteile abzubauen, was den ersten Teil des Buches ausmacht.

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Dinge Eigenschaften kontingenterweise exemplifizieren können. Da Akzidenzien und ihre Subjekte Totalitäten bilden, entsteht ein Weltbild von Seinsschichten, in dem die Kontingenz in der Notwendigkeit aufgehoben wird.

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*Joseph M. Bocheński*

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*Logische Studien zur Summa Theologiae I, qq.2-11*

Joseph M. Bocheński analysiert und formalisiert in diesem Buch sowohl Texte, die die fünf Wege des Thomas von Aquin behandelt, als auch die, die das Wesen Gottes zum Thema haben.

Bocheński prüft die Beweise im Einzelnen und kommt zu dem Fazit, dass nur die *secunda via* schlüssig ist. Er ist von der Qualität der Texte über das Wesen Gottes beeindruckt und rekonstruiert auch sie in der axiomatischen Form, in der sie Thomas verfasst hatte. Zum Schluss geht Bocheński auch auf die Kritik Kants am kosmologischen Gottesbeweis ein und kritisiert diese Kritik. Er versucht zu zeigen, dass Kant Materialien für seine These, nach der der kosmologische Gottesbeweis auf dem ontologischen beruht, ausgerechnet in der *Summa Theologiae* hätte finden können. Bocheńskis Untersuchungen orientierten sich direkt am Text, sind originell und wenden die mathematische Logik in sachgerechter und behutsamer Weise an. Damit fördern sie eine Reihe neuer Ergebnisse und Einsichten zu Tage. Sie sind zudem selbst das Ergebnis einer lebenslangen Beschäftigung mit diesem Thema, das der Autor am Ende seines Lebens noch einmal überdenken und präsentieren wollte. Das ist ihm in Gestalt dieses Buches auch gelungen.

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*Christina Schneider*

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